Handout 14. Dynamic Models

In which you learn how to estimate short and long-run effects and how to test for and deal with the important issue of stationarity in time series data

Common in time series work to try and include lags of explanatory (and dependent) variables in a regression in order to account for belief that the influence of a variable could extend beyond the period in which any change occurred.

$$Y_t = a + b_0X_t + b_1X_{t-1} + u_t$$

(This is called a "distributed lag model of order 1" – since includes variables lagged at most by one period).

The inclusion of lags turns the model from a static one into a dynamic one

Why do this?

1. Technology

   Takes time to change method of production. Eg supply of a factor may change only with a lag following a shock to the production process.

2. Institutions

   The effects of changes in monetary and fiscal policy may take several periods to work through the economy, (multiplier effects)

3. Inertia

   Individuals take time adjusting behaviour

The use of lags allows us to distinguish between the effects of **Long Run and Short Run Multipliers**

Given

$$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

Then the coefficient b_0 is said to be the short-run multiplier effect, since it captures the immediate effect of any change in X on Y at time t

$$b_0 = dY_t/dX_t$$

$$\Rightarrow \quad dY_t = b_0 * dX_t$$

ie Y changes by b_0 times the amount of the change in X

It follows that

$$b_1 = dY_t/dX_{t-1}$$

is the effect on Y at time t of a change in X at time t-1

and

$$b_j = dY_t/dX_{t-j}$$

is the effect on Y at time t of a change in X at time t-j

If X were increased by ΔX in every period
(eg a new higher level of government spending), then in

Period 1

$$\Delta Y = b_0 \Delta X$$

Period 2

$$\Delta Y = b_0 \Delta X + b_1 \Delta X = (b_0 + b_1)\, \Delta X$$

:

Period j

$$\Delta Y = b_0 \Delta X + b_1 \Delta X + \ldots.\ b_j \Delta X$$
$$= (b_0 + b_1 + \ldots.. + b_j)\, \Delta X$$

so $\Delta Y/\Delta X = (b_0 + b_1 + \ldots.. + b_j)$

This sum of all the b coefficients is the **long-run multiplier** effect of a permanent change in the value of X

N.B. 1

When introduce lags this assumes that not just the current value of the X variable is uncorrelated with the residual, but also all past values of X **beyond** the lags already included in the model

$$E(u_t / X_t) = 0$$

and

$$E(u_t / X_{t-1}) = 0$$

… $E(u_t / X_{t-k}) = 0$

… $E(u_t / X_{t-k-s}) = 0$

(which changes the definition of exogeneity a little and ensures that the lagged values included in the original model comprise all the possible non-zero dynamic effects of X)

If we assume that the residuals are also uncorrelated with all **future** values of X this is called **strict exogeneity**

$E(u_t / X_{t+k+s} \ldots X_t \ldots X_{t-k-s}) = 0$

and there may be estimation techniques other than OLS that can be used to estimate dynamic causal effects)

N.B. 2

Can also show that can estimate long run multiplier **and** its standard error directly from the specification

$$Y_t = d_{const} + d_0 \Delta X_t + d_1 \Delta X_{t-1} + \ldots.\ d_j \Delta X_{t-j+1} + d_{j-1} X_{t-j}$$

Where $d_0 = b_0$

$d_1 = b_0 + b_1$

…

$d_2 = b_0 + b_1 + b_2$ etc

and coefficient on X_{t-j} is the long-run multiplier over the whole period

Example: The data set *lagdata.dta* contains quarterly information on a firm's investment, (ie), measured in £ and its revenue (cashf) over 24 years. The idea is to estimate the following relationship.

$$Invest_t = a + b_0*Cashflow_t + u_t$$

To do this read the data in

u lagdata

Then set up time series data in Stata , "time" is the variable in the data set which denotes the period in which the observations on the dependent and explanatory variable was taken. Use the following command.

```
tsset time

Stata responds with

time variable:  time, 1 to 140
```

Now regression, holding back the 1st 8 quarters of data

```
. reg ie cashf   if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  1,    94) =  699.28
       Model |  1.7683e+11     1  1.7683e+11           Prob > F      =  0.0000
    Residual |  2.3770e+10    94   252871543           R-squared     =  0.8815
-------------+------------------------------           Adj R-squared =  0.8802
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   15902
------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |      .8333   .0315121     26.44   0.000      .770732    .8958679
       _cons |   -35688.6    6441.89     -5.54   0.000    -48479.12   -22898.07
------------------------------------------------------------------------------

. bgtest, lag(1)
Breusch-Godfrey LM statistic:  80.03151  Chi-sq( 1)  P-value =  3.7e-19
```

With no lags on cashflow, short-run and long-run multiplier are the same

$$d\ invest/\ d\ cashflow = b_0 = 0.83$$

so a £1 increase in revenue generates an immediate (and permanent) 83 pence increase in investment

Note value of Breusch-Godfrey test indicates that there seems to be (1st order) autocorrelation in the data so standard errors are wrongly estimated, (but coefficients are unbiased).

How many lags to include?

1. Data Mining
– increase the number of lags sequentially until the lagged values start to become insignificant

$$Y_t = a + b_0X_t + u_t$$
$$Y_t = a + b_0X_t + b_1X_{t-1} + u_t$$

$$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

Problems:
- may be a limited number of observations in the data set (quite likely if working with annual time series data) which means "degrees of freedom" problems start to set in

Since T-k ↓ by 1 for every lag added to the model and since variance of OLS coefficient estimates is calculated as

$$Var(\hat{\beta}) = \frac{s_u^2}{T * Var(X)} = \frac{\sum \hat{u}^2 / T-k}{T * Var(X)}$$

the standard error of OLS estimate gets larger as T-k ↓

Could be important for statistical inference (is a variable significant or not)

- More lags ↑ risk of multicolinearity, which again increases standard errors and reduces precision of OLS estimates

$$Var(\hat{\beta}_1) = \frac{s^2}{N * Var(X)} * \frac{1}{1 - r^2_{X_t X_{t-s}}}$$

Example: introduce cashflow lagged one quarter as additional explanatory variable in investment model

$$Invest_t = a + b_0*Cashflow_t \; b_1*Cashflow_{t-1} + u_t$$

```
To generate lags, sort the data by the time variable

sort time
gen cash1=cashflow[_n-1]            /* lags cashflow by 1 period */
```

```
. regdw ie cashf   cash1 if time>8 & time<105

  Source |       SS       df       MS                  Number of obs =      96
 ---------+------------------------------               F(  2,    93) =  392.41
   Model |  1.7934e+11     2  8.9672e+10               Prob > F      =  0.0000
Residual |  2.1252e+10    93   228518067               R-squared     =  0.8941
---------+------------------------------               Adj R-squared =  0.8918
   Total |  2.0060e+11    95  2.1115e+09               Root MSE      =   15117
------------------------------------------------------------------------------
      ie |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
cashflow |   .2309931   .1839125      1.256   0.212      -.1342206    .5962068
   cash1 |   .6089191   .1834484      3.319   0.001       .2446269    .9732114
   _cons |  -35721.57   6123.845     -5.833   0.000      -47882.31   -23560.83

Durbin-Watson Statistic =  .1145408
```

When introduce lags on cashflow into the model, short-run and long-run multiplier are **not** the same

Short-run multiplier = d invest/ d cashflow = b_0 (as before) = 0.23

Long-run multiplier = $b_0 + b_1$ = 0.23 + 0.61 = 0.84

So long-run effect seems to be much larger than short-run effect, (but agrees with estimate from first regression without lags)

Note while the introduction of lags can sometimes reduce autocorrelation, in this case still appear to get autocorrelation in model.

Now do data mining and add 6 lags to the model

```
. reg ie cashf   cash1-cash6  if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  7,    88) =  131.17
       Model |  1.8305e+11     7  2.6150e+10           Prob > F      =  0.0000
    Residual |  1.7544e+10    88   199368717           R-squared     =  0.9125
-------------+------------------------------           Adj R-squared =  0.9056
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   14120
------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |   .2572755   .1786268     1.44   0.153    -.0977078    .6122588
       cash1 |    .141559   .2771014     0.51   0.611    -.4091217    .6922398
       cash2 |   .1152186   .2787114     0.41   0.680    -.4386617    .6690988
       cash3 |   .0837524   .2780237     0.30   0.764    -.4687612     .636266
       cash4 |   .0501196   .2789373     0.18   0.858    -.5042097    .6044489
       cash5 |   .1307876   .2789578     0.47   0.640    -.4235823    .6851575
       cash6 |    .092952   .1783286     0.52   0.604    -.2614386    .4473426
       _cons |  -39319.27   5783.234    -6.80   0.000    -50812.23   -27826.31
------------------------------------------------------------------------------

. bgtest, lag(1)
Breusch-Godfrey LM statistic:  90.22796  Chi-sq( 1)  P-value =  2.1e-21
```

Note however there are big changes to estimated coefficients and standard errors when add several lagged cashflow variables, (because of multicolinearity)

Now **none** of cashflow variables is significant and coefficient on $cash_{t-1}$ has changed considerably.

As a result it is harder to estimate the short and long run multipliers accurately

Short-run multiplier now risen to 0.26
Long-run multiplier = 0.25 + 0.14 + 0.12 + 0.08 + 0.05 + 0.13 + 0.09 = 0.86

(Again very similar to first estimate)

Check multicolinearity by looking at correlation coefficients.

```
. corr ie cashf   cash1-cash6  if time>8 & time<105
(obs=96)

         |       ie cashflow    cash1    cash2    cash3    cash4    cash5
---------+---------------------------------------------------------------
      ie |   1.0000
cashflow |   0.9389   1.0000
   cash1 |   0.9446   0.9866   1.0000
   cash2 |   0.9445   0.9668   0.9861   1.0000
   cash3 |   0.9406   0.9481   0.9658   0.9858   1.0000
   cash4 |   0.9348   0.9332   0.9466   0.9655   0.9857   1.0000
   cash5 |   0.9287   0.9223   0.9332   0.9476   0.9664   0.9861   1.0000
   cash6 |   0.9190   0.9128   0.9226   0.9335   0.9478   0.9662   0.9861

Can see all cash flow variables are highly colinear
```

2. Koyck Transformation

rather than estimate a model with a large number of lags can transform data into a more “parsimonious” form

Given a dynamic model

(1) $$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

Assume effect of a change in X recedes over time by an amount λ each period and that this is reflected in size of coefficients such that

(2) $$b_k = b_0\lambda^k \qquad 0 < \lambda < 1$$

(λ is a fraction so raising fraction to a power ensures effect gets smaller as lag length k increases. The larger the value of λ the slower the speed of adjustment)

We know the long-run multiplier is given by

$$b_0 + b_1 + b_2 + \ldots + b_k = b_0/1 - \lambda$$

(ie the sum of an infinite series with constant of multiplication λ)

Sub. (2) into (1)

(3) $$Y_t = a + b_0X_t + b_0\lambda X_{t-1} + b_2\lambda^2 X_{t-2} + \ldots + b_k \lambda^k X_{t-k} + u_t$$

If (3) is true at time t it is also true at time t-1, so

(4) $$Y_{t-1} = a + b_0X_{t-1} + b_0\lambda X_{t-2} + b_0\lambda^2 X_{t-3} + \ldots + b_0 \lambda^k X_{t-k-1} + u_t$$

multiply (4) by λ

(5) $$\lambda Y_{t-1} = \lambda a + b0\lambda Xt_{-1} + b_0\lambda^2 X_{t-2} + b_0\lambda^3 X_{t-3} + \ldots b_0 \lambda^{k+1} X_{t-k-1} + \lambda u_t$$

(3) – (5)

$$Y_t - \lambda Y_{t-1} = a - \lambda a + b_0X_t + u_t - \lambda u_t$$

or

(6) $$Y_t = (a - \lambda a) + b_0X_t + \lambda Y_{t-1} + v_t \quad (\text{where } v_t = u_t - \lambda u_t)$$

this is called the Koyck transformation and hence the coefficient on the lagged dependent variable gives an estimate of λ with which can estimate long run multiplier $b_0/1- \lambda$ given estimate on lagged value of X

- more parsimonious (fewer coefficients to estimate) and so less chance of multicolinearity

Example: attempt Koyck transformation so that can represent above more parsimoniously(is as current level of cashflow and a lagged dependent variable)

```
. g ie1=ie[_n-1]


. reg ie cashf ie1  if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  2,    93) = 7778.11
       Model |  1.9940e+11     2  9.9702e+10           Prob > F      =  0.0000
    Residual |  1.1921e+09    93  12818348.5           R-squared     =  0.9941
-------------+------------------------------           Adj R-squared =  0.9939
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =  3580.3

------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |   .1260826   .0182838     6.90   0.000     .0897747    .1623906
         ie1 |   .8801663    .020972    41.97   0.000       .83852    .9218125
       _cons |  -8096.317   1592.425    -5.08   0.000    -11258.56   -4934.076
------------------------------------------------------------------------------
```

Coefficient on cashflow is short-run multiplier estimate, (0.13)

Coefficient on ie1 is estimate of rate of decay of cashflow effect on investment over time (λ) = 0.88

So long-run multiplier is $b/(1-\lambda) = 0.13/(1-0.88) = 1.08$

Note doesn't solve the other problem of autocorrelation since

```
. bgtest, lag(1)
Breusch-Godfrey LM statistic:  19.23427  Chi-sq( 1)  P-value =  1.2e-05
```

Since estimated chi-squared greater than critical value, reject null of no autocorrelation, conclude that positive autocorrelation exists.

Important

1. Unfortunately if there is a lagged dependent variable **and** autocorrelation then OLS makes all estimates **inconsistent** (biased) and hence estimates of short and long-run multiplier are wrong. Need to instrument lagged dependent variable if want to use this set-up in this particular example (no need if no autocorrelation

2. Note also that the Koyck transformation does not give a standard error around the estimate of the long-run multiplier. As an alternative you can estimate the standard error using the following specification

$$Y_t = a + d_0\Delta X_t + d_1\Delta X_{t-1} + d_2\Delta X_{t-2} + \ldots.. + d_k\Delta X_{t-k+1} + b_k X_{t-k} + u_t \qquad (1)$$

And the coefficient on X_{t-k} ie b_k is the cumulative long-run multiplier estimate (Proof left to problem sets):

To generate lags of changes

```
gen cash1=cashflow[_n-1]          /* lags cashflow by 1 period */
gen cash2=cashflow[_n-2]          /* lags cashflow by 2 periods */

gen dc=cashflow-cash1             /* 1st period – 2nd period difference */
gen dc1=cash1-cash2

reg ie dc dc1 cash2 if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  3,    92) =  283.06
       Model |  1.8099e+11     3  6.0330e+10           Prob > F      =  0.0000
    Residual |  1.9608e+10    92   213130254           R-squared     =  0.9023
-------------+------------------------------           Adj R-squared =  0.8991
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   14599

------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          dc |   .3445294   .1822557     1.89   0.062    -.0174461    .7065048
         dc1 |   .3544745   .1771636     2.00   0.048     .0026124    .7063366
       cash2 |   .8530673   .0293783    29.04   0.000     .7947195     .911415
       _cons |  -37537.75   5950.109    -6.31   0.000    -49355.18   -25720.32
```

and the coefficient on cash2 gives the long run multiplier (compare with estimate above using Koyck transformation)

Ultimately whether you can sensibly include lags of either the dependent or explanatory variables in a regression also depends on whether the time series data that you are analysing are **stationary**

A variable is said to be (weakly) stationary if

1) its mean
2) its variance
3) its autocovariance Cov(Yt, Yt-s) where s$\neq$ t

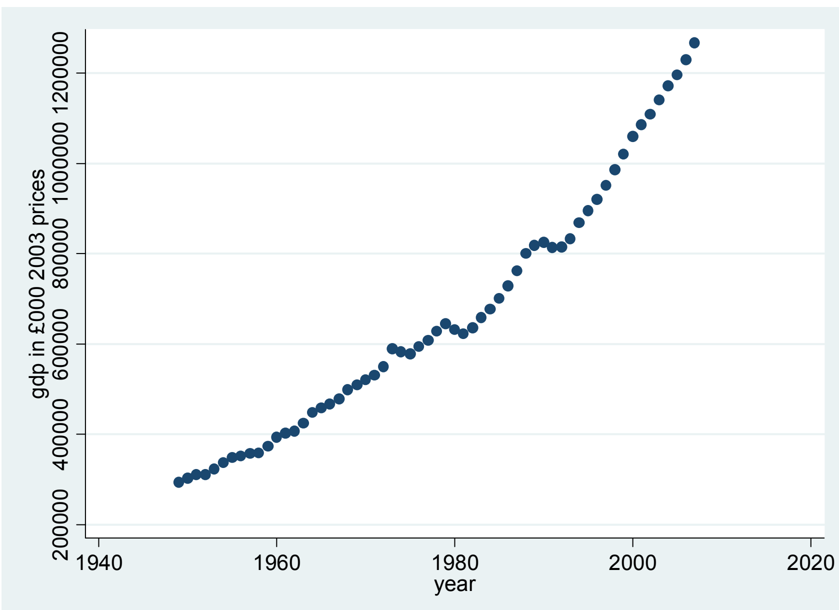
do not change over time

Stationarity is need if the Gauss-Markov conditions need for unbiased, efficient OLS estimation are to be met by time series data

(Essentially any variable that is **trended** is unlikely to be stationary)

Example: the data set stationary.dta allows you to plot real GDP over time

```
two (scatter gdp year)
```

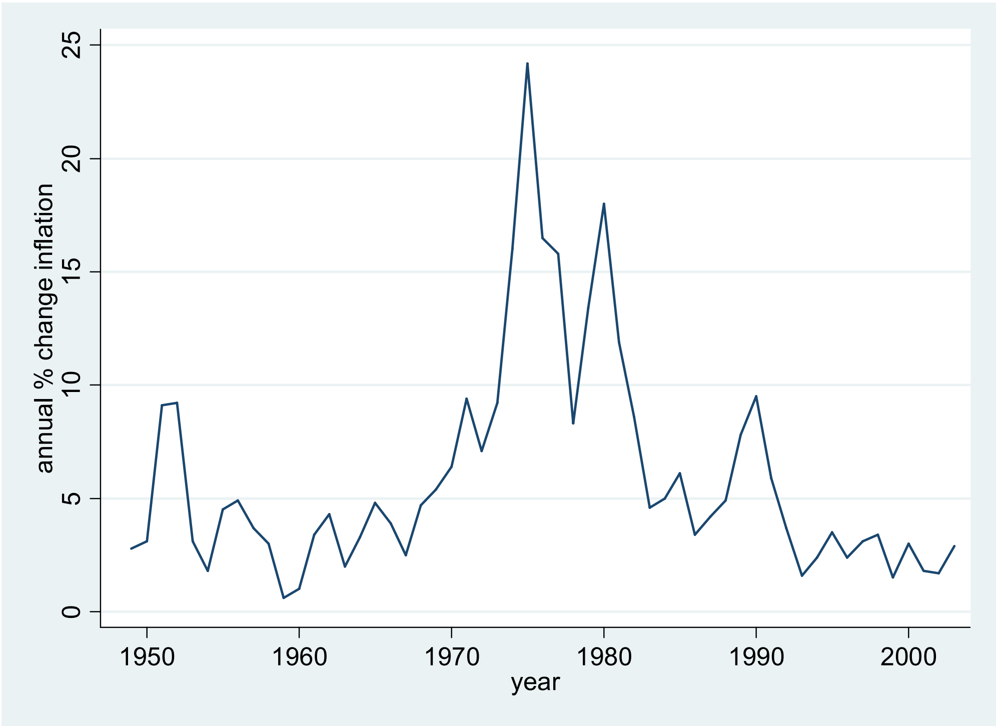


GDP displays a distinct upward trend and so is unlikely to be stationary. Neither its mean value or its variance are stable over time

```
su gdp if year<1980

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         gdp |        31    451016.3      109771     293576     644491

. su gdp if year>=1980

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         gdp |        28    900660.8    196878.9     622722    1266397
```

Some series are already stationary if there is no obvious trend and some sort of reversion to a long run value. The UK inflation rate is one example

two (line drpi year)



In general just looking at the time series of a variable will not be enough to judge whether the variable is stationary or not (though it is good practice to graph the series anyway)

Note that if a variable is stationary then its values are **persistent**. This means that the level of the variable at some point in the (relatively distant) past continues to influence the level of the variable today. This could be important for policy making.

The simplest way of modelling a non-stationary process is the **random walk**

$$Y_t = Y_{t-1} + e_t$$

ie the value of Y today equals last period's value plus an unpredictable random error

(similar to the AR(1) model used for autocorrelation but with the coefficient set to "1")

This means that the best forecast of this period's level is last period's level,
(this model is often used to test the efficiency of stock market behaviour)
Since many series (like GDP) have an obvious trend, can adapt this model to allow for a movement ("drift") in one direction or the other by adding a constant term

$$Y_t = b_0 + Y_{t-1} + e_t$$

This is a **random walk with drift**
and the best forecast of this period's level is now is last period's value **plus** a positive constant (more realistic model of GDP)

Consequences
Can show that if variables are NOT stationary then

1. OLS estimates of coefficient on lagged dependent variable are biased toward zero

2. OLS t values are biased

3. Can lead to **spurious regression** – variables appear to be related but this is because both are trended. If take trend out would not be.

4. Durbin Watson values are biased down (toward 0)

Example: Suppose you decide to regress United States inflation rate on the level of British GDP. There should, in truth, be very little relationship between the two (it is difficult to argue how British GDP could really affect US inflation)

If you regress US inflation rates on UK GDP for the period 1956-1979

```
. u gdp
. reg usinf gdp if year<1980 & quarter==1
      Source |       SS       df       MS              Number of obs =      24
-------------+------------------------------           F(  1,    22) =   50.81
       Model |  156.605437     1  156.605437           Prob > F      =  0.0000
    Residual |  67.8141518    22  3.08246144           R-squared     =  0.6978
-------------+------------------------------           Adj R-squared =  0.6841
       Total |  224.419589    23  9.75737343           Root MSE      =  1.7557
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |    .0001402   .0000197     7.13   0.000     .0000994     .000181
       _cons |   -9.352343   1.945736    -4.81   0.000    -13.38755   -5.317133
```

which appears to suggest a significant positive (causal) relationship between the two. The R^2 is also very high

and if you regress US inflation rates on UK GDP for the period 1980-2002

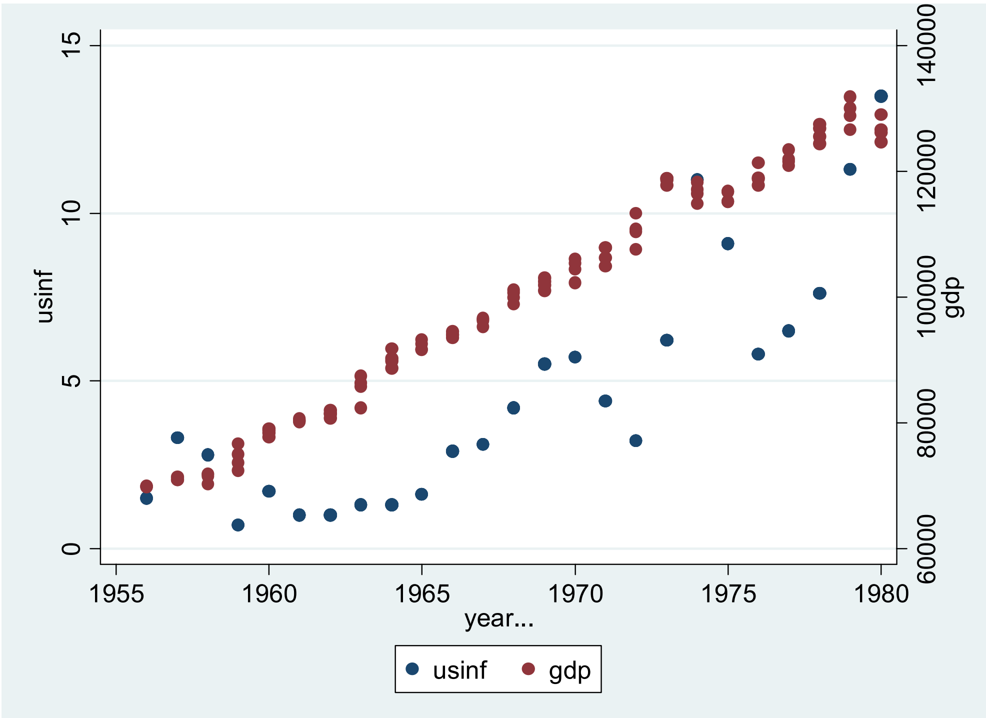
```
. reg usinf gdp if year>=1980 & quarter==1
      Source |       SS       df       MS              Number of obs =      23
-------------+------------------------------           F(  1,    21) =   11.48
       Model |  59.6216433     1  59.6216433           Prob > F      =  0.0028
    Residual |  109.033142    21  5.19205437           R-squared     =  0.3535
-------------+------------------------------           Adj R-squared =  0.3227
       Total |  168.654785    22  7.66612659           Root MSE      =  2.2786
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   -.0000589   .0000174    -3.39   0.003     -.000095   -.0000227
       _cons |    13.77226   2.904938     4.74   0.000     7.731107    19.81341
```

this now gives a significant negative relationship and the R^2 is much lower

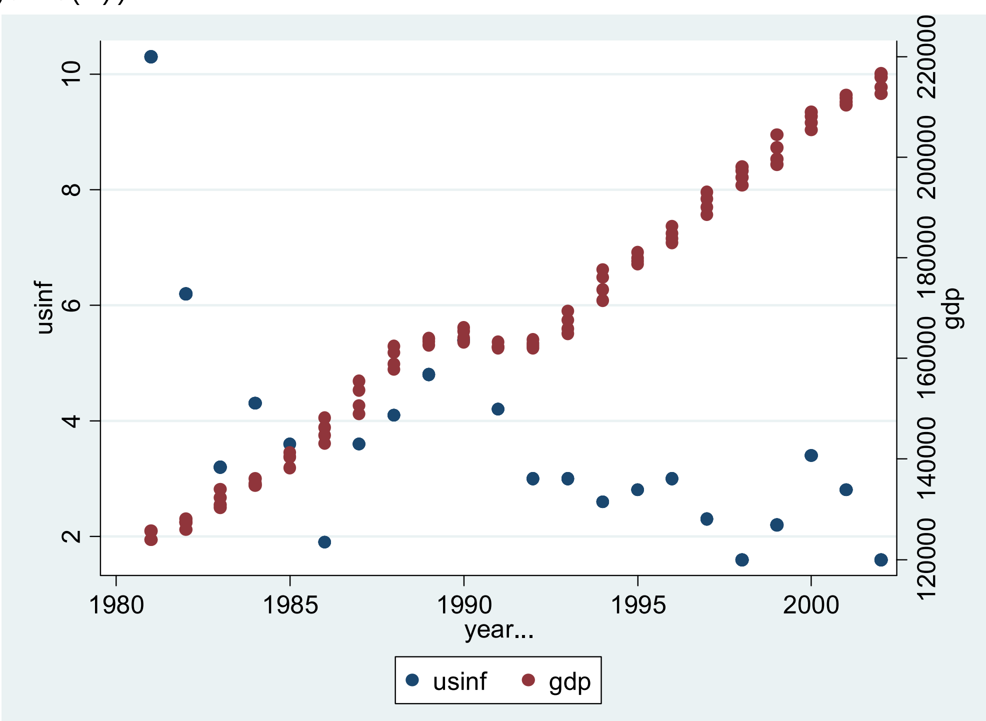
In truth it is hard to believe that UK GDP has any real effect on US inflation rates. The reason why there appears to be a significant relation is because both variables are

trended upward in the 1st period and the regression picks up the common (but unrelated) trends. This is spurious regression

```
twoway (scatter usinf year if year<=1980) (scatter gdp year if year<=1980,
yaxis(2))
```




```
twoway (scatter usinf year if year>1980) (scatter gdp year if year>1980,
yaxis(2))
```



What to do?

- Make the variables stationary and OLS will be OK

Often the easiest way to do this is by **differencing** the data (ie taking last period's value away from this period's value)

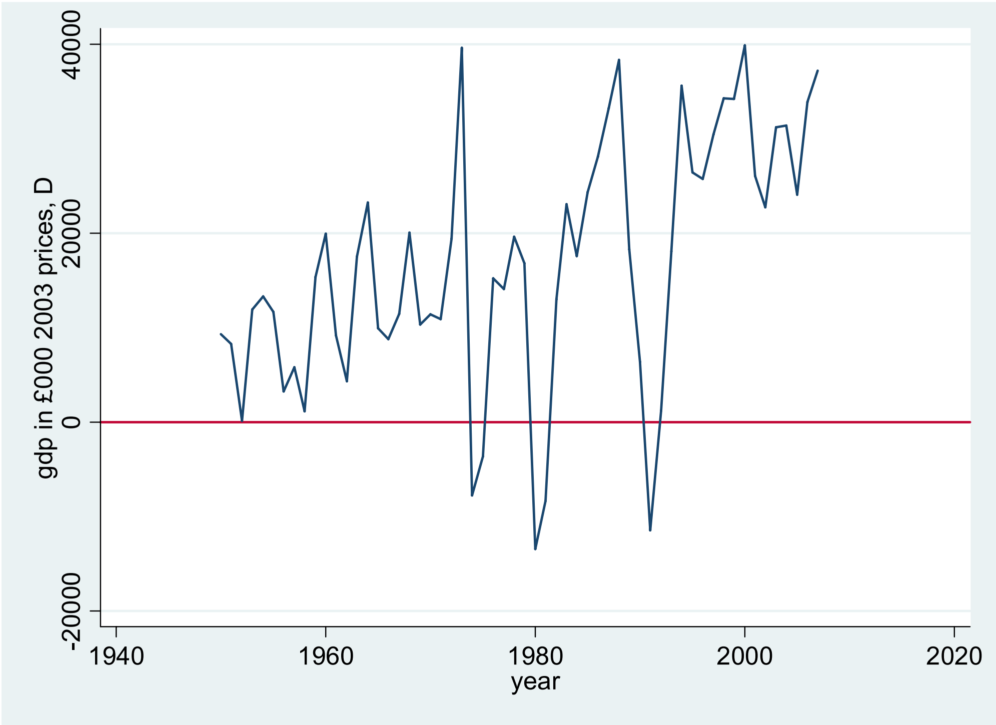
Eg

If $Y_t = Y_{t-1} + e_t$ is non-stationary

Then $Y_t - Y_{t-1} = \Delta Y_t = e_t$

Should be stationary ie random and not trended (since the differenced variable is just equal to the random error term – which has no trend or systematic behaviour)

Example: The change in gdp looks more likely to be stationary.



By inspection it seems there is no trend in the difference of GDP over time (and hence the mean and variance look reasonably stable over time)

Note: Sometimes **taking the (natural) log** of a series can make the standard deviation of the log of the series constant. If the series is exponential (as sometimes is GDP) then the log of the series will be linear and the standard deviation of the log across sub-periods will be constant (since the log of a series changes by the same amount in each sub-period)

Not always easy to tell by looking at a series whether it is a random walk (non-stationary) or not.

Need to test this formally

Detection

Given $Y_t = Y_{t-1} + e_t$ is non-stationary (1)

But $Y_t = \mathbf{b}Y_{t-1} + e_t$ is stationary if b<1 (2)

The test of stationarity is a test of whether b=1

In practice can re-write (2) as

$$Y_t - Y_{t-1} = \mathbf{b}Y_{t-1} - Y_{t-1} + e_t$$

(subtract Yt-1 from both sides of (2))

$$\Delta Y_t = (b-1)\, Y_{t-1} + e_t$$

$$\Delta Y_t = g\, Y_{t-1} + e_t \qquad (3)$$

and test whether g= b-1 = 0 (if g=0 then b=1)

If so, the data follow a random walk and so the variable is non-stationary

Turns out that the critical values of this test differ from the normal t test critical values (in fact 5% critical value = 1.94 – Dickey Fuller Test and 2.86 if there is a constant in the regression)

So accept null of random walk if g is not significantly different from zero.

Example: To test formally whether the UK house prices are stationary or not

```
. u price_sta

tsset TIME
        time variable:  TIME, 24004 to 24084
                delta:  1 unit

. g dprice=price-price[_n-1]              /* creates 1st difference variable */
(1 missing value generated)

. g d2price=dprice-dprice[_n-1]

. reg  dprice l.price

      Source |       SS       df       MS              Number of obs =      80
-------------+------------------------------           F(  1,    78) =    3.32
       Model |  8932482.25     1  8932482.25           Prob > F      =  0.0724
    Residual |   210035668    78  2692764.98           R-squared     =  0.0408
-------------+------------------------------           Adj R-squared =  0.0285
       Total |   218968150    79  2771748.74           Root MSE      =    1641
------------------------------------------------------------------------------
      dprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       price |
         L1. |  -.0088124   .0048384    -1.82   0.072     -.018445    .0008202
       _cons |   3012.817   869.2151     3.47   0.001     1282.343    4743.291
------------------------------------------------------------------------------
```

```
. dfuller price

Dickey-Fuller test for unit root                   Number of obs   =        80
                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.821            -3.538            -2.906            -2.588
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.3699
```

Since estimated t value < Dickey-Fuller critical value (2.86) can't reject null that null that g= 0 (and b=1) and so original series (ie the level, not the change in prices follows a random walk. So conclude that house prices are a non-stationary series

If we repeat the test for the 1st difference in prices (ie the change in prices)

```
. reg  d2price l.dprice
      Source |       SS       df       MS              Number of obs =      79
-------------+------------------------------           F(  1,    77) =   29.84
       Model |  67875874.8     1  67875874.8           Prob > F      =  0.0000
    Residual |   175154796    77  2274737.61           R-squared     =  0.2793
-------------+------------------------------           Adj R-squared =  0.2699
       Total |   243030671    78  3115777.83           Root MSE      =  1508.2
------------------------------------------------------------------------------
     d2price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      dprice |
         L1. |  -.5589261   .1023204    -5.46   0.000    -.7626721   -.3551801
       _cons |   810.6236   225.3341     3.60   0.001     361.9261    1259.321
------------------------------------------------------------------------------
```

Since estimated t value now > Dickey-Fuller critical value (2.86) **reject** null that g= 0 (and b=1) and so new series (ie the change in, not the level of prices) is a stationary series

Should therefore use the change in prices rather than the level of prices in any OLS estimation (same test should be applied to any other variables used in a regression)

Note: stata will do (a variant of) this test automatically – note that the critical values are different since stata includes lagged values of the dependent variable in the test (the augmented Dickey Fuller test)

```
. dfuller dprice, regress
 Dickey-Fuller test for unit root                   Number of obs   =        79
                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.463            -3.539            -2.907            -2.588
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
------------------------------------------------------------------------------
    D.dprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      dprice |
         L1. |  -.5589261   .1023204    -5.46   0.000    -.7626721   -.3551801
       _cons |   810.6236   225.3341     3.60   0.001     361.9261    1259.321
------------------------------------------------------------------------------
```

p value is <.05 so again reject null that g= 0 (and b=1)

COINTEGRATION

If economic data have to be differenced in order to avoid the problems of spurious regressions. In particular it becomes harder to interpret the coefficients from a differenced equation as anything other than the effect of the *change in X* on the *change* in Y

$$\Delta Y_t = b_0 + b_1 \Delta X_t + u_t$$

When we might like to find the effect on the *level* of Y of a change in the *level* of X

$$Y_t = b_0 + b_1 X_t + u_t$$

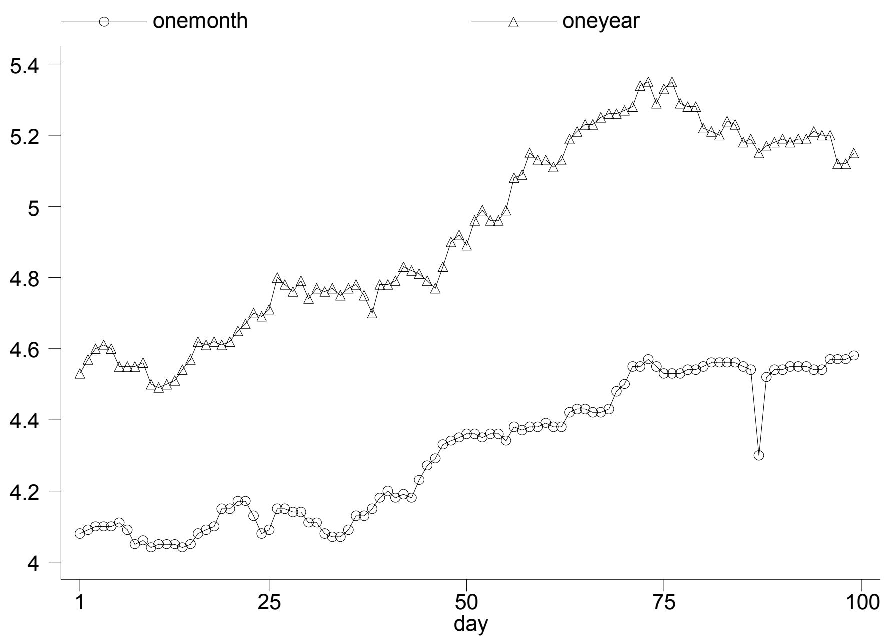
(If we can only estimate a model in changes it is hard to find out the long run (steady state) effect when X is constant ie $\Delta X=0$)

However if a long-run relationship exists, the 2 variables are said to be **cointegrated.** The trick is to try and tease out the long-run relationship.

If 2 non-stationary variables have a ***common trend*** then we can net it out by using some function of the difference in the 2 series

$Y_t - \delta X_t$ where δ is some constant

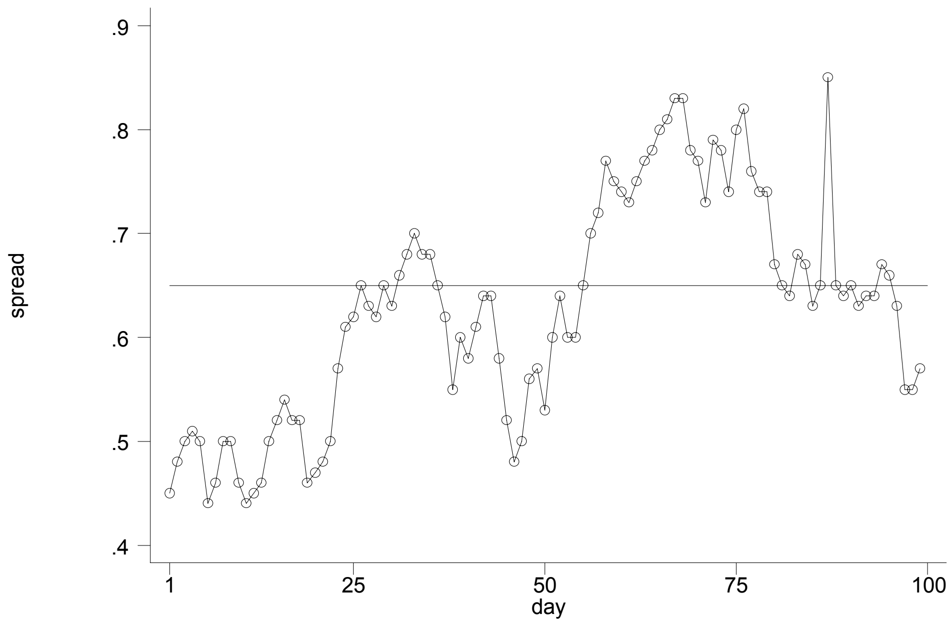
variables with a common trend are also said to be cointegrated



This is a graph of the level of UK one-year and one-month interest rates. Long-term interest rates have a higher return to compensate for the longer lending period. Can see over time however that the interest rates tend to move in the same direction

The difference between the 2 series is called the *spread*

Would not expect the spread to be trended over any significant length of time (otherwise it would be worth shifting all funds into the more favourable asset)



It appears that the spread is centred around .65 % age points over time

In this case the 2 interest rates have a common trend and so are cointegrated

If using time series variables you must ensure that the series are either stationary or that the variables are cointegrated. If not then the problem of spurious regressions holds.

The difference in the series will therefore be stationary and therefore can include this in a regression

$$\Delta Y_t = b_0 + b_1 \Delta X_t + b_2(Y_t - \delta X_t) \qquad u_t$$

The usefulness of this procedure lies in the fact that the coefficient b_2 is called the **error correction effect** and it can be used to assess the extent of movement of y to its long run equilibrium value following a change in X