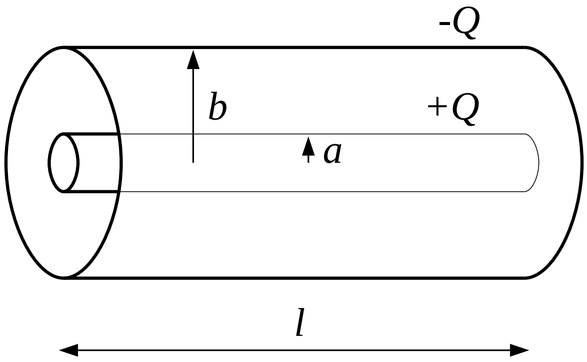
PH 2420 *Electromagnetism*
Worksheet 1

CAPACITORS AND CYLINDRICAL SYMMETRY

Write your name here to get your first mark ....



A cylindrical capacitor is constructed from two long concentric cylindrical conductors of length l and radius a and b (where $a<b$).

There is a charge of $+Q$ on the inner conductor and $-Q$ on the outer conductor.

First, a few questions to introduce the idea of surface charge density.

a) What is the surface area of the inner tube? Area = ...

b) Since the charge on the inner tube is $+Q$, and knowing its area, the areal charge density, the charge per unit area σ is $\sigma = +$ ....

c) The units of σ are units : ....

For completeness, let us consider the outer tube as well.

d) The surface area of the outer tube is Area = ...

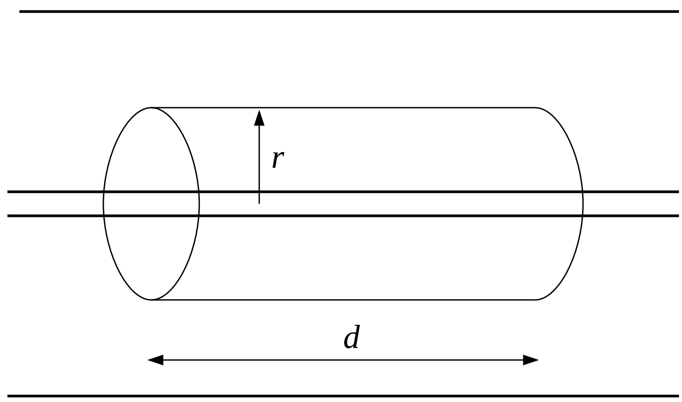
e) The surface charge density σ on the outer tube is $\sigma = -$ ....

Now we shall consider the electric field E between the cylinders.
Note that it is because of the symmetry of the system that we can use Gauss's law − but of course the cylindrical symmetry only applies so long as we can ignore the effects at the ends, that is, if the cylinders are long: $l \gg b$.

f) By considering the symmetry of the system, what is the direction of ***E*** between the cylinders? Direction is: ....

g) What is the component of ***E*** along the length of the cylinder? Axial component of ***E*** is: ....

To evaluate the electric field E we construct a 'gaussian' cylinder of radius r and length d placed concentrically between the inner and the outer cylinder.



h) Gauss's law tells us that the flux of ***E*** through the surface of the cylinder, $\int \boldsymbol{E}.d\boldsymbol{a}$ is given by $1/\varepsilon_0$ times what? ....

i) In terms of the total charge Q, what is the charge enclosed by the 'gaussian' cylinder? Charge = ...

j) How much flux passes through the ends of the gaussian cylinder? Flux through ends = ...

k) What is the surface area of the gaussian cylinder through which the lines of ***E*** penetrate? Surface area = ...

l) Since the ***E*** field is is perpendicular to the curved surface of the cylinder, and it is constant over the surface of the cylinder, Gauss's law tells us that

$$E \times \text{area} = \text{charge enclosed} / \varepsilon_0.$$

So at a radius r what is the magnitude of ***E***? $E(r)$ = ...

At this stage you should have obtained $E(r)=Q/2\pi\varepsilon_0 rl$. *If not then you must go back and check − if necessary refer to your notes on Gauss's law.*

m) The potential difference between two points is given by the work done in moving *what* between the points? Moving ....

n) The work done in moving a charge q parallel to an electric field ***E***, is given by the integral $W = -q\int$ ....

o) What is the - sign doing in this expression?
The - sign is here because we must do work by applying the force .... the electric force experienced by the charge.

p) The *potential difference V* between the two cylindrical conductors of radius *a* and *b* is the work done in moving a unit charge from one cylinder to the other:

$$V \quad = \quad -\int_a^b E(r)\ \mathrm{d}r \ .$$

Since the dependence of *E* on *r* is known, this integral can be evaluated. Find the expression for *V*:

Do your working in the box below

V = ...

q) The *capacitance* is given in terms of the potential difference *V* and the charge *Q*. The expression is: C = ...

r) So what is the expression for the capacitance of the cylindrical capacitor, in terms of its dimensions etc.? C = ...

Some interesting observations:

In the expression for *C* we have the factor $2\pi\varepsilon_0$. This is typical of systems with *cylindrical* symmetry, just as one finds $4\pi\varepsilon_0$ for systems with *spherical* symmetre − e.g. the field or potential of a point charge.

Note also that the *logarithm* in the expression for *C* is also a characteristic of cylindrical symmetry.

The capacitance is proportional to ε_0. For a dielectric medium the electric permittivity is enhanced by the dielectric constant. We see that the capacitance is enhanced by the same factor.