PH2420 Electromagnetism
Problem Sheet 5

Note: Your work must be clearly presented; marks will be deducted for assignments that are scrappy and difficult to follow.

1. Write down the (microscopic) Maxwell equations for **E** and **B** in a vacuum. By taking the curl of one of the curl equations show how the Maxwell equations imply the existence of electromagnetic waves. What is the speed of propagation of these waves in terms of μ_0 and ε_0?

2. By writing oscillating voltages and currents in the complex form

$$V(t) = ve^{i\omega t}, \qquad I(t) = ie^{i\omega t}$$

show that the voltages and currents in the primary and the secondary of a transformer may be expressed as

$$v_1 = i\omega(L_1 i_1 + M i_2)$$
$$v_2 = i\omega(M i_1 + L_2 i_2).$$

If an impedance Z_2 is connected across the secondary of the transformer demonstrate that the impedance seen at the primary is

$$Z = i\omega L_1 - \frac{\omega^2 M^2}{Z_2 - i\omega L_2}.$$

Discuss the special cases where Z_2 is a) a short-circuit, and b) an open-circuit

3. Why are the equations for div**B** and curl**E** *not* modified in the presence of dielectric and magnetic media?

4. Under what circumstances is the trajectory of a charged particle in an electromagnetic field a closed circle? What is the radius of the circle?

5. From the point of view of electromagnetism matter can be regarded as an assembly of bound charges and currents.

Show how the microscopic time-independent field equation

$$\text{div}\mathbf{E} = (\rho_{\text{free}} + \rho_{\text{bound}})/\varepsilon_0$$

may be re-expressed by the introduction of the macroscopic quantity **P** where

$$\rho_{\text{bound}} = -\text{div}\mathbf{P}$$

to give a macroscopic field equation in terms of the field vector **D**.

Write down this macroscopic field vector.

Discuss the microscopic interpretation and origin of the vector **P**.