PH2420 Electromagnetism
Problem Sheet 4

Note: Your work must be clearly presented; marks will be deducted for assignments that are scrappy and difficult to follow.

1. A particle of charge q is moving with a velocity **v** in an electric field **E** and a magnetic field **B**. Under what circumstances will the charge experience no net force?

2. In a *ballistic galvanometer* the coil assembly has significant inertia so that for a short pulse of current the deflection is proportional to the charge passed through the coil. This can form the basis of a system for measuring magnetic field B. A wire loop of area a is connected to the galvanometer. The loop is suddenly moved out of the magnetic field. Using Faraday's law of electromagnetic induction show that the deflection of the galvanometer will be proportional to Ba/R where R is the resistance of the galvanometer coil.

3. a) How does the capacitance of a parallel plate capacitor (in a vacuum) depend on the area and separation of the plates?
b) A parallel plate capacitor in a vacuum carries a charge Q. Show that this charge is related to the electric field E between the plates by
$$Q = \varepsilon_0 AE$$
where A is the area of the plates.
c) Show that an electric current I flowing into one plate and out of the other will cause a varying electric field according to
$$I = \varepsilon_0 A \frac{\partial E}{\partial t}.$$
d) In terms of Ampère's law, which may be expressed as
$$\oint \mathbf{B}.\mathrm{d}l = \mu_0 I,$$
explain how the above result indicates that a time-varying electric field will produce a magnetic field.

4. Show that the vector calculus identity div curl ≡ 0 implies that Ampère's law $\mathrm{curl}\mathbf{B} = \mu_0 \mathbf{J}$ and the equation of continuity $\mathrm{div}\mathbf{J} = -\partial\rho/\partial t$ are incompatible. By modifying Ampère's law to $\mathrm{curl}\mathbf{B} = \mu_0 \mathbf{J} + \mathbf{F}$, show that the div curl identity and the equation of continuity are satisfied if
$$\mathrm{div}\mathbf{F} = \mu_0 \frac{\partial \rho}{\partial t}.$$
Use the Gauss's law expression $\mathrm{div}\mathbf{E} = \rho/\varepsilon_0$ to show that **F** must satisfy
$$\mathbf{F} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$
Discuss the significance of this result.

5. An electric field wave propagating in the **k** direction may be written as
$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{\mathrm{i}(\mathbf{k}.\mathbf{r} - \omega t)}.$$
By writing **E**, **k** and **r** in terms of their Cartesian components (or otherwise), show that
$$\mathrm{div}\mathbf{E} = \mathrm{i}\mathbf{k}.\mathbf{E} \quad \text{and} \quad \mathrm{curl}\mathbf{E} = \mathrm{i}\mathbf{k} \times \mathbf{E}.$$
Use this result (and the corresponding result for the **B** field) to show that the **E** field, the **B** field and the direction of propagation are mutually perpendicular. (You should recall how the Maxwell equations were used to show the existence of electromagnetic waves).