PH2420 Electromagnetism
Problem Sheet 3

Note: Your work must be clearly presented; marks will be deducted for assignments that are scrappy and difficult to follow.

1. Assuming the Poisson equation

$$\nabla^2 V = -\rho/\varepsilon_0$$

show that the electric field in a cavity within a conductor is zero.

Hint: a) Explain why ρ is zero in the cavity; then the equation reduces to Laplace's equation.
b) Explain why the electric potential V is constant on the surface of the cavity.
c) Show that Laplace's equation then implies that V = const. within the cavity.
d) Show that this means $E = 0$ within the cavity.
(*Explain clearly each step of your argument*)

2. The model for the discussion of the resistivity of metals treated in the lectures can be described in a slightly different way by incorporating the scattering term into the electron equation of motion

$$m\ddot{x} + \frac{2m}{\tau}\dot{x} = qE$$

where x is interpreted as a mean quantity.

The DC conductivity is found by considering the *steady state* solution when E is constant. Show that in this case the above equation of motion leads to a mean velocity $qE\tau/2m$ and that this results in the familiar expression for the conductivity

$$\sigma = \frac{Nq^2\tau}{2m}.$$

Now consider the case where an *oscillating* electric field is applied, by writing E as $E_0 e^{\mathrm{i}\omega t}$.
Solve the equation of motion in this case and show that this leads to a *complex* AC conductivity varying with frequency as

$$\sigma(\omega) = \frac{\sigma(o)}{1 + \mathrm{i}\omega\tau/2}.$$

Sketch the real and the imaginary parts of the conductivity as a function of frequency.

3. An anisotropic two-dimensional conductor has a conductivity tensor given by

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} \text{ siemens.}$$

What is the magnitude and direction of the current caused by an electric field $\mathbf{E} = 2\hat{\mathbf{x}} + 3\hat{\mathbf{y}}$ volts per metre?

There are two directions for which the current flows parallel to the applied field. Find these directions (*hint*: think of eigenvectors and eigenvalues). What is the magnitude of the conductivity in these directions?

4. An overhead power cable, 10 m high, is carrying a current of 10^3 ampères. What is the resultant magnetic field at ground level? In what direction does this field point?

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