PH2420 Electromagnetism
Problem Sheet 2

Note: Your work must be clearly presented; marks will be deducted for assignments that are scrappy and difficult to follow.

1. Four charges, each of magnitude $+Q$ are placed at the corners of a square of side a. Obtain an expression for the electric potential at the centre of the square and at the mid point of one side. What is the direction of the electric field at the centre of the square and at the mid point of the side (symmetry arguments will help you)?

2. Application of Gauss's law
   a) State *Gauss's law* relating the flux of an electric field through a closed surface to the total electric charge enclosed.
   b) Define *electric potential* and show that the potential V at a distance r from a point charge Q is given by
   $$V = \frac{Q}{4\pi\varepsilon_0 r}.$$
   c) A sphere of radius R contains a uniform charge density ρ. How much charge is contained in a concentric sphere of radius a (where a is less than R)?
   d) Using this result together with Gauss's law, how does the electric field vary with radius within the sphere?
   e) And how does the electric potential vary with radius within the sphere?
   f) How does the electric field vary with radius outside the sphere?
   g) How does the electric potential vary with radius outside the sphere?
   h) Sketch the variation of E and V with distance from the centre of the sphere.
   Hint: In applying Gauss's law to an imaginary spherical volume, obviously you only consider the charge *enclosed* by the volume.

3. The 'classical radius' of the electron
   According to one theory, the mass m of the electron arises simply as a consequence of the electrostatic energy of its electric field, through Einstein's famous expression $\mathcal{E} = mc^2$, where $\mathcal{E}$ is the field energy and c is the speed of light. The energy of the electron's field is found by integrating the field energy density U over all space. If a is the radius of the electron and e is its charge, show that the field energy $\mathcal{E}$ is given by
   $$\mathcal{E} = \frac{e^2}{8\pi\varepsilon_0 a}.$$
   In this picture the electron cannot be regarded as a point charge. Why?
   Show that this theory gives the radius of the electron as
   $$a = \frac{e^2}{8\pi\varepsilon_0 mc^2}.$$
   Calculate the value of the electron's radius.

4. Show that
   $$\operatorname{grad}\frac{1}{r} = -\frac{\mathbf{r}}{r^3}.$$
   Hint: work in Cartesian co-ordinates and write $r = \sqrt{x^2 + y^2 + z^2}$. Then you can differentiate $1/r$ and assemble the components.