X ELECTRIC FIELDS AND MATTER
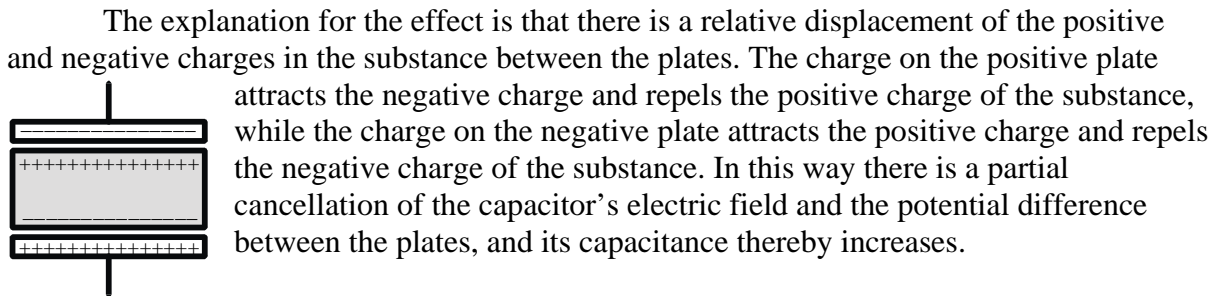
10.1 Dielectrics and dipoles

It is an experimentally observed fact that if you put some (insulating) matter between the plates of a capacitor then the capacitance *increases*. Since the capacitance is given by the ratio of charge to voltage

$$C = Q/V$$

this means that when matter is inserted between the plates

Q increases when V is held constant

or V decreases when Q is held constant.

The first case would apply to a capacitor connected to a battery, and the second case applies to an electrically isolated capacitor. We will consider the second.

The explanation for the effect is that there is a relative displacement of the positive and negative charges in the substance between the plates. The charge on the positive plate

 attracts the negative charge and repels the positive charge of the substance, while the charge on the negative plate attracts the positive charge and repels the negative charge of the substance. In this way there is a partial cancellation of the capacitor's electric field and the potential difference between the plates, and its capacitance thereby increases.

The (insulating) material between the plates is called a *dielectric*. The enhancement of the capacitance of a capacitor by a dielectric is thus a consequence of the *polarisation* of the dielectric. So to understand the nature of dielectrics, we must be familiar with the properties of polarised matter.

The simplest model for polarisation is a *dipole*. This is an equal positive and negative point charge separated by a small displacement.

There are *two* origins for dipoles in matter:

1) Permanent dipoles, for example NaCl

 There is a *permanent* dipole due to the asymmetric charge distribution associated with the ionic bond of the NaCl "molecule". The sodium ion loses an electron and the chlorine ion gains it. While the charge distribution is asymmetric, the entire assembly remains electrically neutral.

2) Induced dipoles, for example O_2 (in an electric field)
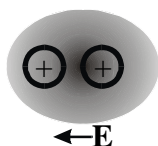
 Here an applied electric field *induces* a polarisation by pulling the positive charges one way and the negative charges the other way. This can happen in individual atoms and in covalently bonded molecules, as O_2, where the bonding electron orbits become distorted by the electric field.

We shall see that dipoles can explain *most* of the electrical properties of insulators.

We *define* the electric dipole moment **p** as

$$\mathbf{p} = q\,\mathbf{l} \tag{10.1}$$

where **p** points along **l**, from $-q$ to $+q$. In the next section we will start to explore the properties of electric dipoles. However, here we will consider the simple application of Gauss's law to a dipole.

Gauss's law tells us that the flux of **E** through a closed surface is proportional to the total charge in the enclosed volume. Considering a surface enclosing a dipole, we see that the total emergent flux is zero. To zeroth order the dipole has no electrical effect: all the lines of **E** leaving the positive charge terminate on the negative one; there are no free lines. But what is important is the first non-vanishing order. Although there is no net flux, in some places the lines of **E** go in, and in other places they go out – as we shall see.
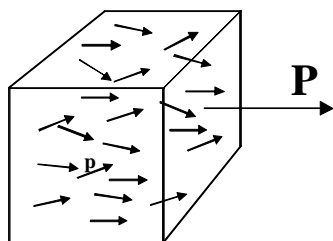
The bulk electric behaviour of matter is described in terms of the electric polarisation vector **P**. This is defined as the total dipole moment per unit volume.

$$\mathbf{P} = N\langle \mathbf{p}\rangle \tag{10.2}$$

where N is the number of dipoles of moment **P**, per unit volume.

10.2 Electric potential of a dipole

As a prelude to finding the electric field of a dipole we shall calculate the electric potential at the "field point" F by adding the potentials from the two source charges.
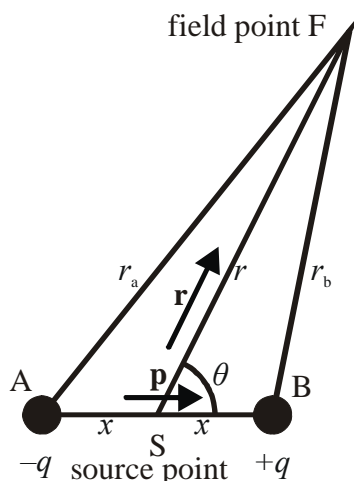
The separation of the charges is $2x$, which we had denoted by l. The magnitude of the dipole moment in this case is then

$$p = 2xq.$$

The potential at point F is given by

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_{\mathrm{b}}} - \frac{1}{r_{\mathrm{a}}}\right)$$

We are considering the potential at a distance r from the dipole very much greater than the separation $2x$ between the charges. After all, in practice r is a macroscopic distance while x is a microscopic dimension. In this case we see that the distances r_{a} and r_{b} are very similar. To the extent that they are equal we see that there is no potential, and thus no field. We must thus consider specifically the difference between (the reciprocals) of these distances. However, since x/r is such a small quantity, we may consider an expansion in this small quantity, keeping only the leading term.

Expressions for r_a and r_b may be written in terms of the distance r and the angle θ by the use of the cosine rule:

$$r_a^2 = r^2 + x^2 + 2xr\cos\theta$$
$$r_b^2 = r^2 + x^2 - 2xr\cos\theta$$

so taking the square root we obtain r_a as

$$r_a = r\left\{1 + \frac{2x}{r}\cos\theta + \left(\frac{x}{r}\right)^2\right\}^{1/2}$$

and its reciprocal:

$$\frac{1}{r_a} = \frac{1}{r}\left\{1 + \frac{2x}{r}\cos\theta + \left(\frac{x}{r}\right)^2\right\}^{-1/2}$$

We may now expand through the use of the binomial theorem:

$$\frac{1}{r_a} = \frac{1}{r}\left\{1 - \frac{x}{r}\cos\theta - \frac{1}{2}\left(\frac{x}{r}\right)^2 + \dots\right\}$$

with a similar expression for r_b by changing the sign of the cosine:

$$\frac{1}{r_b} = \frac{1}{r}\left\{1 + \frac{x}{r}\cos\theta - \frac{1}{2}\left(\frac{x}{r}\right)^2 + \dots\right\}$$

From these we find the difference in the reciprocal distances as

$$\frac{1}{r_b} - \frac{1}{r_a} = \frac{2x}{r^2}\cos\theta + \dots$$

and so to leading order in the small quantity x/r the electric potential at point F due to the dipole at point S is given by

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2x}{r^2}\cos\theta .$$

But since the dipole moment p is simply $2xq$, the potential may be expressed as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \qquad (10.3)$$

Note that for the dipole $V \sim 1/r^2$, compared with $V \sim 1/r$ for a point charge (electric *mono*pole). We shall see that this gives an **E** field that varies as $1/r^3$ for the dipole compared with $1/r^2$ for the monopole. The dipole field thus falls off much *faster*; it has a shorter range.

Taking account of the direction of the dipole, since

$$p\cos\theta = \mathbf{p}.\hat{\mathbf{r}} = \mathbf{p}.\mathbf{r}/r$$
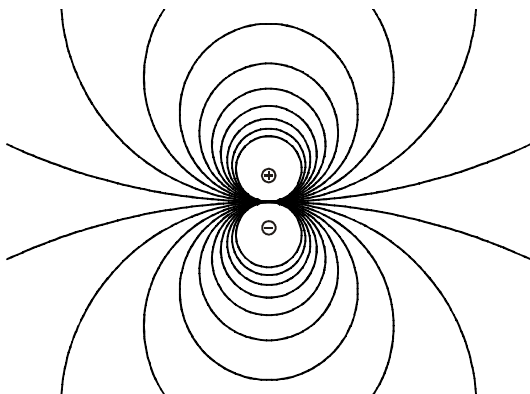
we can write the dipolar potential in vector form as

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}.\hat{\mathbf{r}}}{r^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}.\mathbf{r}}{r^3}.$$

(10.4)

It is interesting to note from our definition and properties of the grad function that since the change in a quantity V on undergoing a displacement d$\mathbf{r}$ is given by Equation (3.24):

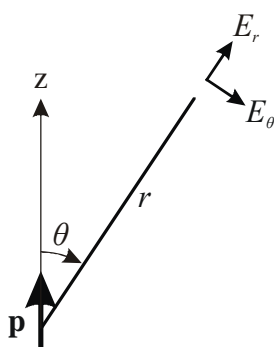$$\mathrm{d}V \;=\; \mathrm{d}\mathbf{r}\,.\,\mathrm{grad}(V),$$

the potential of the dipole may be written in the compact form:

$$V = \frac{1}{4\pi\varepsilon_0}\mathbf{p}.\,\mathrm{grad}\left(\frac{1}{r}\right).$$

(10.5)

The figure shows equipotential lines for an electric dipole. The equipotential lines for a single charge are concentric circles. Observe how the equipotentials around the charges of the dipole are distorted.

10.3 Electric field of a dipole

Since the electric field is a vector, we must decide what co-ordinate system to use when describing $\mathbf{E}$ in terms of its components. While rectangular co-ordinates are the most familiar, the symmetry of this particular system makes polar co-ordinates much more convenient to use. Recall that the electric field is given as the negative gradient of the electrostatic potential:

$$\mathbf{E} \;=\; -\mathrm{grad}V.$$

We are familiar with the expression for the components of the gradient in rectangular co-ordinates; these were given in Equation (3.23). You should have encountered the gradient in other co-ordinate systems from your maths courses. In polar co-ordinates the components of grad$V$ are given by (there is no dependence on φ)

$$E_r = -\frac{\partial V}{\partial r}, \qquad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \qquad\qquad (10.6)$$

so that using V for the dipole, from Equation (10.3), we find

$$E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}, \qquad E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3} \qquad\qquad (10.7)$$
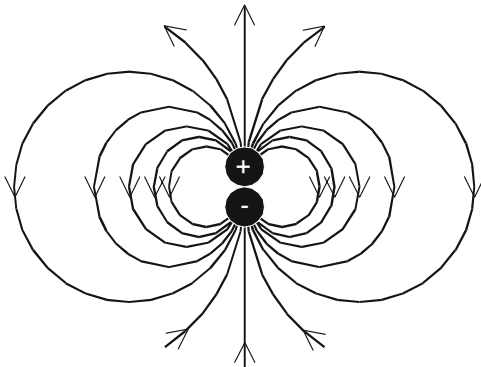

The diagram shows the lines of **E** surrounding an electric dipole. Observe the characteristic shape of the dipole field. Although there is much cancellation of the fields from the two charges, it is not complete: thus the dipole field. The almost-cancellation may be clearly seen by superimposing the fields from the two charges. The lines from one charge point out while those from the other point in. Where the lines cross there is almost complete cancellation, with the remnant field essentially perpendicular. Thus the dipole field lines follow the Moiré curves of the intersections of the lines.
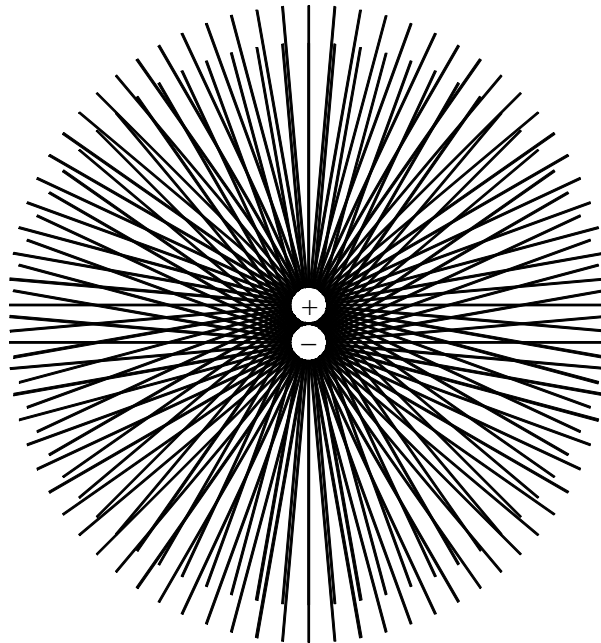
10.4 Dipole in a uniform E field
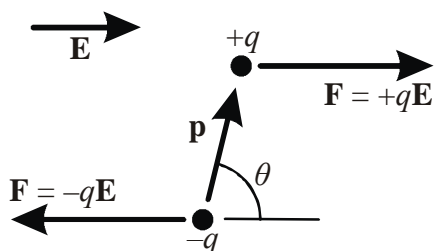
In a uniform electric field the forces on the two charges constituting the dipole will be equal and opposite, although not co-linear. Thus there will be no net force on the dipole but there will be a couple (a torque).

For a dipole **p** in a uniform **E** field we conclude:

$$\text{Force} = 0 \tag{10.8}$$

$$\text{Couple} = E\,q\,\mathrm{d}l\,\sin\theta = \mathbf{p} \times \mathbf{E}. \tag{10.9}$$

We shall now calculate the potential energy of a dipole in a uniform electric field. The potential energy of the dipole is the work done in bringing it to its particular orientation from some reference state. The torque on the dipole due to the external **E** field has magnitude $pE\sin\theta$ and it acts in the clockwise direction. So to rotate the dipole we must apply an anticlockwise torque: in the direction of increasing θ. And the work done is the integral of the torque with respect to θ:

$$\text{Work done} = \int \text{Torque}\,\mathrm{d}\theta$$

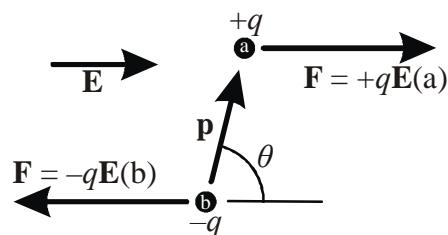$$= Ep \int \sin\theta\,\mathrm{d}\theta$$

$$= -Ep\cos\theta.$$

We have used the *indefinite* integral as we do not want to have the complication of considering the initial reference orientation; the above result is correct to within the addition of a constant. It may be seen that the constant is zero when the reference state is taken as that when the dipole is pointing *perpendicular* to the field. Then the potential energy has the simple form:

$$V = -\mathbf{E}.\mathbf{p}\,. \tag{10.10}$$

(Note that *V* here is the potential *energy* and not the electric potential.)

10.5 Dipole in a non-uniform E field

In a uniform field there was no net force on a dipole because the force on one charge was exactly balanced by the force on the other. This was because both charges experienced the same electric field. If, however, the field were non-uniform then the two charges would be in slightly different fields and so they would experience slightly different forces. We may write the net force on the dipole as

$$\mathbf{F} = q\,\{\,\mathbf{E}(a) - \mathbf{E}(b)\,\}.$$

Now $\mathbf{E}(b)$ may be related to $\mathbf{E}(a)$ by a Taylor expansion

$$\mathbf{E}(b) = \mathbf{E}(a) - \left\{ \frac{\partial \mathbf{E}}{\partial x} x + \frac{\partial \mathbf{E}}{\partial y} y + \frac{\partial \mathbf{E}}{\partial z} z \right\}$$

where (x, y, z) are the components of the vector from point a to point b.

Thus we have for the force

$$\mathbf{F} = q \left\{ \frac{\partial \mathbf{E}}{\partial x} x + \frac{\partial \mathbf{E}}{\partial y} y + \frac{\partial \mathbf{E}}{\partial z} z \right\}$$
$$= p_x \frac{\partial \mathbf{E}}{\partial x} + p_y \frac{\partial \mathbf{E}}{\partial y} + p_z \frac{\partial \mathbf{E}}{\partial z}.$$

This is an interesting mathematical structure. In this form it *has* to be written in terms of the components of the dipole moment $\mathbf{p}$; it cannot be expressed in coordinate-independent form involving $\mathbf{p}$ and our familiar div, grad and curl. However we note that the x component of the force can be written as

$$F_x = \mathbf{p}.\operatorname{grad} E_x$$

and similarly for the y and z components. Thus

$$F_x = \mathbf{p}.\operatorname{grad} E_x$$
$$F_y = \mathbf{p}.\operatorname{grad} E_y \qquad (10.11)$$
$$F_z = \mathbf{p}.\operatorname{grad} E_z$$

These may be conveniently combined together in the following symbolic manner:

$$\mathbf{F} = (\mathbf{p}.\operatorname{grad})\mathbf{E}. \qquad (10.12)$$

We must regard Eq. (10.11) as the definition of the $\mathbf{p}$.grad operator. Interestingly we have found that this very structure actually appears in the expressions for grad($\mathbf{A}.\mathbf{B}$) and curl($\mathbf{A} \times \mathbf{B}$) in Chapter 3.

10.6 Free and bound charge

We now come to a most important concept in the consideration of the electrical properties of matter. Our fundamental viewpoint is that matter is simply a collection of charges and currents. However, the charges that constitute matter are different from the charges that are responsible for charging an insulator or which flow as current in a conductor. We are led to the idea of *bound* charge. This is the charge within neutral matter which, when displaced, is responsible for its polarisation: causing for the dipole moments of the constituent atoms or molecules.

Recall that in Section 10.1 we introduced the *polarisation* $\mathbf{P}$ of a macroscopic body: the dipole moment per unit volume. So if there are N dipoles per unit volume, and each dipole has a moment $\mathbf{p}$ then the polarisation is given by

$$\mathbf{P} = N \langle \mathbf{p} \rangle. \qquad (10.13)$$

10.7 Bound surface charge density

If an insulator has a polarisation then the displaced bound charge produces a net positive charge at one end and a negative charge at the other end. The dipole moment of the strip is is given by its volume times **P**:
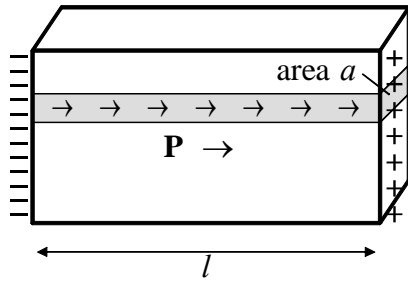
$$\text{dipole moment} = a\, l\, \mathbf{P}.$$

But this strip may be regarded as a single dipole: charges $+q$ and $-q$ separated by a distance l. Thus

$$lq = alP$$

so that the equivalent charge on the ends of the strip are $+aP$ and $-aP$. The area of the strip is a so that the bound surface charge density σ_b is simply P. Taking into account the orientation of the surface with respect to the direction of the polarisation the bound charge on the area a may be written as

$$Q_b = \mathbf{P}\cdot\mathbf{a} \tag{10.14}$$

and the surface charge density as

$$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}. \tag{10.15}$$

Here $\hat{\mathbf{n}}$ is the unit vector pointing perpendicularly from the surface.

10.8 Bound volume charge density

If the polarisation of a body is not uniform then there will be a non-zero charge density *within* the body even though as a whole it is electrically neutral. When the body becomes polarised there is an internal movement of bound charge. Applying the arguments of the previous section to a small element within the body, the bound charge flowing out of a volume v across a surface area d$\mathbf{a}$ is given by **P**.d$\mathbf{a}$. So the total charge flowing out of the volume is

$$Q_{\text{out}} = \oiint_{\substack{\text{closed} \\ \text{surface}}} \mathbf{P}.\mathrm{d}\mathbf{a}.$$

But the volume *was* neutral. So the charge remaining in the volume is then $-Q_{\text{out}}$. And this must be equal to the volume integral of the bound charge (volume) density ρ_b:

$$\iiint_{\text{volume}} \rho_b \mathrm{d}v = - \oiint_{\substack{\text{bounding} \\ \text{surface}}} \mathbf{P}.\mathrm{d}\mathbf{a}.$$

The right hand side of this equation may be transformed to a volume integral through the use of the divergence theorem

$$\oiint_{\substack{\text{bounding} \\ \text{surface}}} \mathbf{P}.\mathrm{d}\mathbf{a} = \iiint_{\text{volume}} \operatorname{div}\mathbf{P}\,\mathrm{d}v$$

so that

$$\iiint\limits_{volume} \rho_b \mathrm{d}v = -\iiint\limits_{volume} \mathrm{div}\,\mathbf{P}\,\mathrm{d}v\,.$$

Now the volume of integration is really quite arbitrary. This equation holds irrespective of the precise volume specified, and this can only hold if the arguments of the two integrals are equal. In other words, the bound charge ρ_b is related to $\mathbf{P}$, the polarisation, by

$$\rho_b = -\mathrm{div}\,\mathbf{P}. \tag{10.16}$$

(There are some analogies between this discussion and the derivation of the equation of continuity.)

10.9 Gauss's law including dielectrics

Our statement of Gauss's law, which we may express as

$$\mathrm{div}\,\mathbf{E} = \rho/\varepsilon_0$$

includes the effect of *all* charges. In our treatment of matter we interpret the electrical properties of dielectrics as due to the action of *bound* charges. In other words we are saying that there are two types of charge we must consider: bound charges and free charges. And Gauss's law above refers to all the charges. We thus write it in the form

$$\mathrm{div}\,\mathbf{E} = \rho_{\mathrm{free}}/\varepsilon_0 + \rho_b/\varepsilon_0.$$

But ρ_b is related to $\mathbf{P}$ by Equation (10.16), so that

$$\mathrm{div}\,\mathbf{E} = \rho_{\mathrm{free}}/\varepsilon_0 - \mathrm{div}\,\mathbf{P}/\varepsilon_0.$$

Now the bound charge, or equivalently the polarisation, is something that we do not have control over; it just happens. It can be regarded as an *effect* rather that a *cause*, and as such we can take the div $\mathbf{P}$ over to the left hand side of the equation, writing

$$\mathrm{div}(\varepsilon_0\mathbf{E} + \mathbf{P}) = \rho_{\mathrm{free}}. \tag{10.17}$$

This equation is giving us another way to look at Gausss's law in the presence of dielectrics. Whereas in the absence of dielectrics the (free) electric charge is the source of the $\mathbf{E}$ field, in the presence of a dielectric of polarisation $\mathbf{P}$ the free charge may be regarded as the source of a field $\mathbf{E} + \mathbf{P}/\varepsilon_0$.

Conventionally one defines a new vector field $\mathbf{D}$, called the *electric displacement*:

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}. \tag{10.18}$$

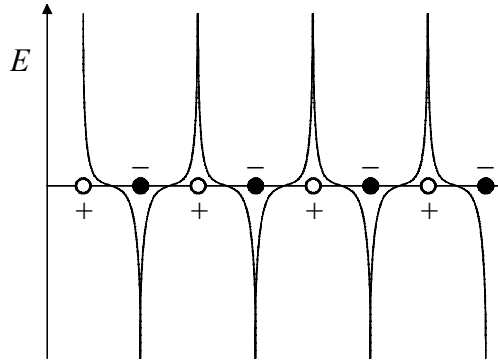In terms of this, Gauss's law in the presence of matter may be written as

$$\mathrm{div}\,\mathbf{D} = \rho_{\mathrm{free}}. \tag{10.19}$$

This is the modification of the Maxwell equation (M1) to include the effects of matter.

Note that only the Maxwell equations with 'sources' will become modified to account for the effects of the bound source, be it charge or current. Thus the div **E** equation is modified but the curl **E** equation will not be. And similarly in the magnetic case the div **B** equation will not be modified, while the curl **B** equation will be.

10.10 Fields in matter

The discussions which led to the introduction of the electric displacement **D** are valid in matter 'on the average'. It is really artificial to talk of the *dipole moment per unit volume*, which was how we defined the polarisation vector **P**. In reality, on the microscopic level, at most places within matter the polarisation is zero, only rising to a large value at the atomic sites. Similarly, the electric field in matter is a very rapidly varying quantity. Since the discussion above only makes sense for continuously varying polarisation **P**, we are really referring to a spatial average over volumes containing a large number of atoms. That means that when using the field equations in their form

$$\text{div } \mathbf{D} \ = \ \rho_{\text{free}}, \quad \text{curl } \mathbf{E} \ = \ -\partial \mathbf{B}/\partial t$$

we are referring to spatially averaged fields; these are two of the *macroscopic* Maxwell equations. However, when we use the field equations in their original form

$$\text{div } \mathbf{E} \ = \ \rho/\varepsilon_0, \quad \text{curl } \mathbf{E} \ = \ -\partial \mathbf{B}/\partial t$$

then we mean the *actual* fields which can vary as rapidly as necessary; these are two of the *microscopic* Maxwell equations.

10.11 Dielectric constant and electric susceptibility

From the phenomenological point of view the macroscopic Maxwell equations have required the use of a new electric field vector, the **D** field. But this presents a complication, because now there are more variables than there are equations. We need another relationship connecting **D** and **E**. In general we will have some functional form

$$\mathbf{D} \ = \ \mathbf{D}(\mathbf{E}) \tag{10.20}$$

which is referred to as a *constitutive relation*. This is a statement about the dielectric medium in which we measure the fields.

For a *linear*, *isotropic homogeneous* dielectric the **E** field and the **D** field will be proportional and parallel. In that case we can write

$$\mathbf{D} \ = \ \varepsilon \mathbf{E} \tag{10.21}$$

where ε is known as the *permittivity* of the dielectric. Alternatively, this can be expressed as

$$\mathbf{D} \ = \ \varepsilon_0 \varepsilon_{\text{r}} \mathbf{E} \tag{10.22}$$

where ε_0 is the permittivity of free space and ε_r is called the *relative permittivity* or *dielectric constant*. Clearly ε_r is dimensionless. For air, ε_r is about 1.0006 at NTP, while for water it is about 81 for static fields. In practice the dielectric constant is found to vary with frequency for oscillatory fields; the polarisation of the atoms can't keep up as the **E** field varies.

The remaining discussions of this section refer specifically to LIH systems. We start with a consideration of what is happening at the *microscopic* level.

For the (induced) dipole moment of an atom or molecule we write

$$\mathbf{p} = \alpha \mathbf{E} \tag{10.23}$$

where α is referred to as the *polarisibility*. This is a relation that holds at the microscopic level. At the *macroscopic* level the polarisation **P** will then be proportional to **E** and we write

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}, \tag{10.24}$$

and χ_e is known as the *electric susceptibility*. The inclusion of ε_0 in the definition means that χ will be dimensionless. The relation between the relative permittivity and the electric susceptibility is easily seen to be
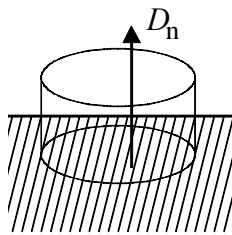
$$\varepsilon_r = 1 + \chi_e . \tag{10.25}$$

10.12 Fields at boundaries

An important problem, when considering dielectrics, is to understand how the electric fields **E** and **D** behave at the boundary between two materials. Apart from the interest in the static case, where one might need to know the field in the vicinity of a surface, this problem is related to the *refraction* of electromagnetic waves at the boundary between different media. It turns out that the boundary conditions on **E** and **D** can be found easily from the relevant Maxwell equations. In the static case and in the absence of free charges, the two 'electric' field equations are
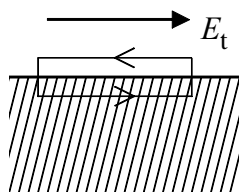
$$\text{div } \mathbf{D} = 0, \quad \text{curl } \mathbf{E} = 0 .$$

Applying div **D** = 0 to the Gaussian pillbox, the total flux of **D** through the surface is zero. If we let the height of the box shrink to zero then the flux of **D** *in* through the bottom must be equal to the flux of **D** *out* through the top. And since the flux of **D** through the surface is given by the normal component of **D** times the area and the areas of the top and the bottom are equal, this means that the normal component of **D** on both sides of the boundary must be equal. In other words the *normal component of* **D** *is continuous at a boundary*. Obviously, the boundary can be between two dielectrics or between one dielectric and free space.

Applying curl **E** = 0 to the loop indicated, we may evaluate the line integral of **E** around the loop. As the height of the loop tends to zero only the tangential parts of the integral contribute. The integral on one side of the boundary cancels with the one on the other side. And since the directions of integration are opposite, it follows that the two line integrals (evaluated in the same direction) must be equal. The lengths are equal and so therefore the components of **E** along the surface must be equal. In other words the *tangential component of* **E** *at a boundary is continuous*.

10.13 Coulomb's law revisited

In a vacuum the force between two charges is given by Coulomb's law. We now investigate how this will be changed in the presence of a dielectric. Clearly we are considering the case where the two charges are immersed in a fluid (liquid or gas) dielectric. In a solid there would be other stresses involved when embedded charges were moved even slightly.

In the vacuum case we went from the inverse square law for **E**:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

to the Gauss's law expression giving the divergence of **E**:

$$\text{div } \mathbf{E} = \rho/\varepsilon_0.$$

Now in the presence of a dielectric medium Gauss's law, in general, becomes rather complicated since the bound charges of the medium must be included in ρ. However, we have seen that the difficulty may be side-stepped by the introduction of the electric displacement vector **D**. We then recover a Gauss's law expression, for **D**, in terms of the *free* charges

$$\text{div } \mathbf{D} = \rho_{\text{free}}.$$

For isotropic media, where the lines of the field will emanate from a charge uniformly in all directions, the expression for div **D** may be inverted, to give

$$D = \frac{Q_{\text{free}}}{4\pi r^2}$$

where the charge Q here is the free charge, and note that there is no ε_0 in the expression. Strictly this inversion relies on curl**D** being zero (spherical symmetry), thus we are assuming the medium is *isotropic*.

The force on a second charge q in the dielectric is given by

$$\mathbf{F} = q\mathbf{E},$$

so that we have to find the **E** field corresponding to the value of **D** given above. We have already assumed the dielectric is isotropic, to obtain the expression for **D**. Making the further assumptions of linearity and homogeneity allows us to relate **E** to **D** through the permittivity or dielectric constant:

$$\mathbf{E} = \mathbf{D}/\varepsilon = \mathbf{D}/\varepsilon_0\varepsilon_r$$
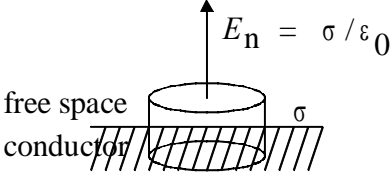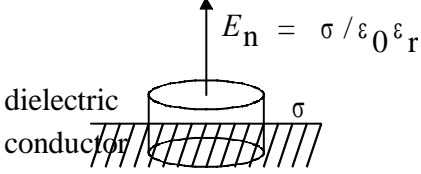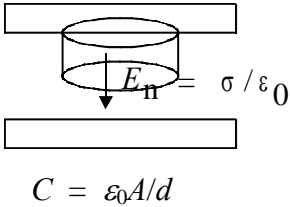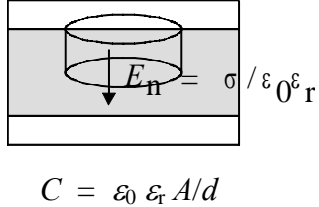
so that the force is given by

$$F = \frac{qQ}{4\pi\varepsilon_0\varepsilon_r r^2} \;.$$

This is identical to the original Coulomb's law, except that now we have the absolute permittivity $\varepsilon = \varepsilon_0\varepsilon_r$ rather than the free space value ε_0. Thus the force is simply reduced, in inverse proportion to the relative permittivity. Note, however, that the simplicity of the analogy is somewhat illusory; the result is only valid for LIH dielectrics.

10.14 Comparison of dielectrics and free-space

Where necessary we assume LIH materials.

	Free space	**Dielectrics**
1)	$\operatorname{div}\mathbf{E} = \rho/\varepsilon_0$	$\operatorname{div}\mathbf{D} = \rho_{\text{free}}$ $\operatorname{div}\mathbf{E} = \rho_{\text{free}}/\varepsilon_0\varepsilon_r$
2)	$\mathbf{E} = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{\hat{\mathbf{r}}}{r^2}$	$\mathbf{E} = \dfrac{Q_{\text{free}}}{4\pi\varepsilon_0\varepsilon_r}\dfrac{\hat{\mathbf{r}}}{r^2}$
3)	$\mathbf{F} = q\mathbf{E}$	$\mathbf{F} = q\mathbf{E}$
4)	$\mathbf{F} = \dfrac{qQ}{4\pi\varepsilon_0}\dfrac{\hat{\mathbf{r}}}{r^2}$	$\mathbf{F} = \dfrac{qQ}{4\pi\varepsilon_0\varepsilon_r}\dfrac{\hat{\mathbf{r}}}{r^2}$

5)

Free space: $E_n = \sigma/\varepsilon_0$ — free space conductor

Dielectrics: $E_n = \sigma/\varepsilon_0\varepsilon_r$ — dielectric conductor

| 6) | $\nabla^2 V = -\rho/\varepsilon_0$ | $\nabla^2 V = -\rho/\varepsilon_0\varepsilon_r$ |

7)

Free space: $E_n = \sigma/\varepsilon_0$ $C = \varepsilon_0 A/d$

Dielectrics: $E_n = \sigma/\varepsilon_0\varepsilon_r$ $C = \varepsilon_0\varepsilon_r A/d$

It is appropriate to make a number of observations about the above table of results. The left hand column, referring to 'free space' describes the behaviour of the *microscopic* field **E** in terms of the sources, which are *all* the charges present. In contrast to this, the right hand column, referring to dielectrics, is describing the *macroscopic* or spatially averaged fields **E** and now **D** as well. The sources are now the (spatially averaged) *free* charges alone. The bound charges are accounted for in the polarisation **P**, which contributes to the **D** field.

Of particular importance is the expression for the electric force

$$\mathbf{F} = q\mathbf{E}.$$

This is true in both the microscopic and the macroscopic picture, although the interpretation is different in the two cases. From the microscopic point of view both **E** and **F** may be very rapidly varying quantities (in the vicinity of the elementary charges which constitute matter). However, from the macroscopic point of view both the force **F** and the electric field **E** are regarded as spatial averages and they have negligible variation on the atomic scale.

When you have completed this chapter you should:

- understand that a dielectric increases the capacitance of a capacitor

- appreciate that this is related to polarisation of the dielectric;

- understand the distinction between permanent and induced electric dipoles;

- appreciate that most of the electrical properties of insulator can be understood in terms of dipoles;

- understand the definition of electric dipole moment;

- appreciate that the field of a dipole results from the almost cancellation of the fields of the positive and negative charges of the dipole;

- be able to calculate the electric potential of a dipole;

- be able to calculate the electric field of a dipole;

- understand that in a uniform electric field a dipole experiences a torque but no force;

- understand the expression for the energy of an electric dipole in an electric field;

- understand that in a non-uniform electric field the dipole will experience a force and be able to calculate this force;

- appreciate the distinction between free and bound charge;

- understand the definition of polarisation of a macroscopic body;

- understand that a polarised body has an equivalent bound surface charge density;

- understand that in a body with non-uniform polarisation, there is an equivalent bound volume charge density;

- be able to calculate the bound volume charge density in terms of the divergence of the polarisation;

- appreciate how Gauss's law may be re-expressed in the presence of dielectrics and the electric displacement;

- be familiar with and understand the definition of the **D** field;

- understand the distinction between microscopic and macroscopic electric fields;

- understand the distinction between the microscopic and the macroscopic field equations;

- know of the definitions of dielectric constant and electric susceptibility;

- understand the concept of a constitutive relation;

- know the meaning of polarisibility;

- understand and be able to perform calculations on the change in **E** and **D** at boundaries between dielectrics;

- know how Coulomb's law is modified in the presence of (LIH) dielectrics;

- appreciate the various modifications of the properties of free space in the presence of dielectrics.