UNIVERSITY OF LONDON

BSc EXAMINATION 1999

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420B: ELECTROMAGNETISM
PH2420A(R): ELECTROMAGNETISM - PAPER FOR RESIT CANDIDATES

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Calculators ARE permitted

PH2420B and PH2420a(R)/50

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^3
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

e = 2.718 π = 3.142 $\log_e 10$ = 2.303

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***

You are advised not to spend more than **40 minutes** answering ***Question One***

1. (a) Calculate the ratio of the electrostatic force to the gravitational force between a proton and an electron. How does this ratio depend on the particle separation? **[4]**

   (b) Describe and explain qualitatively a possible mechanism for the finite electrical conductivity of metals. **[4]**

   (c) Explain the meaning of the statement that an electric field is conservative. When is an electric field not conservative? **[4]**

   (d) Assuming the Poisson equation

   $$\nabla^2 V = -\rho / \varepsilon_0$$

   show that the electric field in a cavity within a conductor is zero. **[4]**

   (e) How is the Ampère defined? Describe how, following from this definition, electric currents can be measured **[4]**

   (f) Show how an arrangement of electric and magnetic fields can be used to select a beam of charged particles moving with a given velocity. **[4]**

PART MARKS

2. (a) State Gauss's law relating the flux of an electric field **E** out of a closed surface to the total electric charge enclosed. **[4]**

(b) A capacitor consists of two parallel conducting plates each of area A separated by a distance d in free space. One plate carries a charge of $+Q$ and the other $-Q$. By using Gauss's law, or otherwise, show that the electric field between the plates has magnitude

$$E = \frac{Q}{\varepsilon_0 A}.$$ **[5]**

(c) The above result implies that the electric field between the plates is *uniform*. Explain carefully the arguments leading to the uniformity of the field, including a discussion of the limitation of such arguments. **[3]**

(d) Define the electric potential V. Show that the potential difference between the plates is given by

$$V = Ed.$$ **[3]**

(e) Hence demonstrate that the potential difference is proportional to the charge on the plates and show that the capacitance C is given by

$$C = \frac{\varepsilon_0 A}{d}.$$ **[3]**

(f) Outline briefly the effect of placing a dielectric material between the plates. **[2]**

PART MARKS

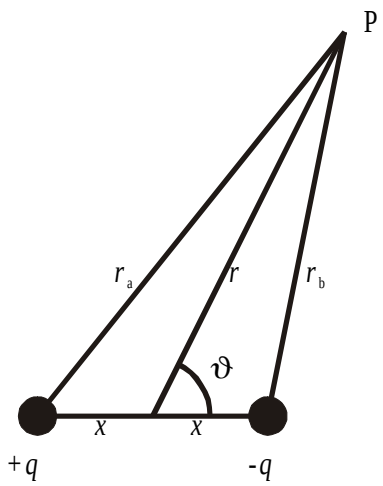
3. (a) Charges $+q$ and $-q$ are separated by a distance $2x$. State the magnitude and the direction of the resultant dipole moment **p**. **[3]**

(b) Show that a dipole experiences no force in a uniform electric field. **[3]**

(c) Show that a dipole moment **p** experiences a torque given by $\mathbf{p} \times \mathbf{E}$ in a uniform electric field **E**. **[4]**

(d) Show that the electric potential at the point P in the figure is given by

$$V = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{r_a} - \frac{1}{r_b}\right\}. \quad \textbf{[2]}$$



(e) Show that for $r >> x$ the electric potential at P may be approximated by

$$V = \frac{p\cos\vartheta}{4\pi\varepsilon_0 r^2}. \quad \textbf{[5]}$$

(f) Explain why a dipole experiences no force in a *uniform* electric field, but it does experience a force in a *non-uniform* field. **[3]**

PART MARKS

4. (a) Show that Ampère's law relating the line integral of the magnetic field **B** around a closed loop to the current *I* passing through the loop,

$$\oint \mathbf{B}.\mathrm{d}\mathbf{l} = \mu_0 I$$

may be expressed in terms of the curl of **B** as

$$\mathrm{curl}\mathbf{B} = \mu_0 \mathbf{J}\ .$$ **[4]**

(b) Demonstrate that there is an inconsistency in Ampère's law when applied to the space between the plates of a capacitor when it is charging up. **[5]**

(c) Since the charge on a capacitor *in vacuo* may be written as

$$Q = \varepsilon_0 AE$$

where *E* is the electric field and *A* is the area of the plates, show that the inconsistency may be removed through the introduction of a *fictitious current i* given by

$$i = \varepsilon_0 A \frac{\partial E}{\partial t}\ .$$ **[6]**

(d) Show that, including the effect of this current, the curl of **B** may be written as

$$\mathrm{curl}\mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\ .$$ **[3]**

(e) In terms of the *electric flux* comment on the parallels with Faraday's law of electromagnetic induction. **[2]**

5. (a) Describe an experiment to verify the *inverse square law* of the Coulomb force to the highest possible accuracy. You should give particular attention to the methods whereby this high accuracy is obtained and errors minimised. **[17]**

(b) Discuss a possible explanation *if* it were to be discovered that the inverse square law were not quite correct. **[3]**

END