UNIVERSITY OF LONDON

BSc EXAMINATION 1998

For Internal Students of

Royal Holloway

<u>DO NOT TURN OVER UNTIL TOLD TO BEGIN</u>

PH2420A: ELECTROMAGNETISM

PH242R: ELECTROMAGNETISM
PAPER FOR RESIT CANDIDATES

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Calculators ARE permitted

PART MARKS

1. ANSWER **ONLY FIVE** sections of *Question One*

   You are advised not to spend more than **40 Minutes** answering *Question One*

   (a) Coulomb's law is:

   $$\boldsymbol{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}}}{r^2}$$

   Define the terms in this equation. A charge $+Q$ is placed at the point $(R, 0, 0)$ and a second charge $-Q$ is placed at the point $(-R, 0, 0)$. Calculate the electric field vector at the point $(0, R, 0)$. [4]

   (b) Define the capacitance C of an isolated conductor. Show that, when such a conductor is at an electric potential V, its electrostatic energy is $\frac{1}{2}CV^2$. [4]

   (c) Ampère's circuital law is given by:

   $$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I$$

   Define carefully the terms in this equation. Use this law to derive the magnetic field at a distance R from a straight, infinitely long current-carrying wire. [4]

   (d) Discuss *briefly* what is meant by hysteresis in relation to ferromagnetic materials. Include in your discussion a labelled sketch of a hysteresis curve. [4]

   (e) What is meant by *self inductance* and *mutual inductance*? Explain *briefly* the relevance of these phenomena to the working of a transformer. [4]

   (f) A charged particle with charge q, mass m and initial velocity $\boldsymbol{v}$ is moving in a region with an electric field $\boldsymbol{E}$. Describe the motion (without detailed proofs) for the cases when

   (i) $\boldsymbol{v}$ is parallel to $\boldsymbol{E}$

   (ii) $\boldsymbol{v}$ is perpendicular to $\boldsymbol{E}$. [4]

PART MARKS

2. Define current density, $\boldsymbol{J}$, conductivity σ and electric field $\boldsymbol{E}$. By considering the case of a cylindrical resistor of length L, cross-sectional area A and resistance R, show that Ohm's Law can be written in terms of these variables as: $\boldsymbol{J} = \sigma \boldsymbol{E}$ **[6]**

   Show carefully that the velocity $\boldsymbol{v}$ of a particle with charge q and mass m, moving under the influence of an electric field $\boldsymbol{E}$ within a solid satisfies the following differential equation:

$$m\frac{d\boldsymbol{v}}{dt} + \frac{2m}{\tau}\boldsymbol{v} = q\boldsymbol{E}$$

   where τ is the relaxation time. **[6]**

   Hence show that the d.c. conductivity of a solid conductor may be written as

$$\sigma = \frac{Nq^2\tau}{2m}$$

   where N is the number of carriers per unit volume. **[5]**

   An a.c. electric field $E = E_0 e^{i\omega t}$ is now applied to the conductor. Derive and sketch an expression for how the amplitude of the resulting a.c. current varies as a function of ω. **[3]**

3. What is meant by an electric dipole moment $\boldsymbol{p}$? Define the polarization $\boldsymbol{P}$ of a dielectric and discuss briefly its origins at an atomic level when an external electric field is present. **[6]**

   Explain qualitatively how *and why* the Maxwell equation in vacuum:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

   is modified in a dielectric with relative permittivity ϵ_r. **[6]**

   Derive the boundary conditions for $\boldsymbol{E}$-fields perpendicular to the boundary between two dielectrics with relative permittivities ϵ_{r1} and ϵ_{r2}. Derive a corresponding boundary condition for the parallel $\boldsymbol{E}$-fields. Which of these boundary conditions would be affected by a uniform surface charge density on the boundary? **[5]**

   A small thin electrically neutral glass rod of length a, volume V and relative permittivity ϵ_r is placed with its centre on the x-axis at $x = D$ where $D >> a$ and with its length along the x-axis. If a charge Q is fixed at the origin, calculate any force or torque acting on the rod. **[3]**

PART MARKS

4. Show how Ampère's circuital law leads directly to the time independent equation:

$$\frac{1}{\mu_0}\nabla \wedge \boldsymbol{B} = \boldsymbol{J}$$

[5]

The Maxwell equation:

$$\frac{1}{\mu_0}\nabla \wedge \boldsymbol{B} = \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J}$$

implies that the term $\epsilon_0 \frac{\partial}{\partial t}\boldsymbol{E}$ has the properties of an electric current density and is called the "displacement current". By considering an imaginary surface which passes between the plates of a parallel plate capacitor and which closes on an imaginary loop around one of the wires to the capacitor, show that the displacement current is necessary to ensure a consistent treatment of the electric and magnetic fields in the vicinity of the capacitor while it is charging up. **[6]**

Draw a sketch showing the electric and magnetic fields within the plates of the capacitor when it is being charged up by a continuous steady current. Label carefully the fields and their directions and indicate qualitatively how the fields vary with time and position. Ignore any edge effects. **[6]**

Given that the vector $\frac{1}{\mu_0}\boldsymbol{E} \wedge \boldsymbol{B}$ gives the flux of energy, what do your diagrams tell you about the flow of energy and the distribution of the energy stored in the capacitor during charging and discharging? **[3]**

PART MARKS

5. Faraday's law of electromagnetic induction can be written as:

$$V = -\frac{d\Phi}{dt}$$

Define all the terms in this expression and explain the origin of the minus sign. **[5]**

Show carefully that Faraday's law leads to the Maxwell equation

$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

[6]

A uniform magnetic field $\boldsymbol{B}$ is set up in the z-direction where $\boldsymbol{B}$ varies linearly with time t according to:

$$\boldsymbol{B} = \beta t \boldsymbol{k}$$

where β is a constant and $\boldsymbol{k}$ is the unit vector in the z-direction. By considering the emf generated in a circular loop of wire of radius R lying in the xy-plane, show that the electric field vector at any point (x, y, z) is given by:

$$\boldsymbol{E} = \frac{\beta}{2}\left(y\boldsymbol{i} - x\boldsymbol{j}\right)$$

[6]

Hence verify that the above Maxwell equation is satisfied in this case. **[3]**

END