UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2006

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420D: ELECTROMAGNETISM

MT324: ELECTROMAGNETIC THEORY

Time Allowed: **TWO** hours

*Answer **QUESTION ONE** and **TWO** other questions*

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators or CASIO fx85MS Calculators are permitted

PH2420D/49

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^{2} kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) State *Faraday's law of electromagnetic induction*. Write this as an equation in terms of the curl of **E**. **[4]**

(b) How is the ampere defined? Show how electric currents may be standardised using the forces they produce. **[4]**

(c) Sketch the *B*–*H* curve of a ferromagnetic material. How may this curve be measured? **[4]**

(d) The magnetic vector potential **A** is related to the magnetic field through the integral relation

$$\iint_{\text{surface}} \mathbf{B}\cdot \mathrm{d}\mathbf{a} = \oint_{\substack{\text{perimeter}\\ \text{of surface}}} \mathbf{A}\cdot \mathrm{d}\mathbf{l}\,.$$

Express this relation in differential form.

How does the magnetic flux penetrating an area depend on the value of **A** at the area's perimeter? **[4]**

(e) Explain when the equation $\mathbf{E} = -\operatorname{grad} V$ is not valid. How is the equation repaired, to be true generally? **[4]**

(f) State Kirchhoff's current and voltage laws. What physical principles do they embody? **[4]**

2. (a) Ampère's law may be written

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \,.$$

Explain, in words, what this means. **[2]**

(b) Show that Ampère's law may be expressed in terms of the curl of **B** as

$$\text{curl}\,\mathbf{B} = \mu_0 \mathbf{J} \,.$$ **[2]**

(c) Demonstrate that there is an inconsistency in Ampère's law when considering the space between a charging capacitor. **[5]**

(d) Since the charge on a capacitor (in a vacuum) may be written as

$$Q = \varepsilon_0 A E$$

where E is the electric field and A is the area of the plates, show that the inconsistency may be removed through the introduction of a fictitious current i (the displacement current) given by

$$i = \varepsilon_0 A \frac{\partial E}{\partial t} \,.$$ **[5]**

(e) Show that including the effect of the displacement current, the curl of **B** may be written as

$$\text{curl}\,\mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \,.$$ **[4]**

(f) Outline, briefly, how the introduction of the displacement current leads to the possibility of electromagnetic waves. **[2]**

3. (a) Define electric current density **J** and electrical conductivity σ. **[2]**

(b) By considering a cylinder of resistive material of length L, cross section A and resistance R, show that Ohm's law can be written $\mathbf{J} = \sigma \mathbf{E}$. **[4]**

(c) Describe a model by which the resistivity of conductors may be understood and give an expression for the conductivity in terms of the parameters of the model. **[10]**

(d) Explain how the conductivity of *anisotropic* media may be described in terms of a *tensor* conductivity. What is the significance of the principal **[4]**

PART MARKS

axes of the conductivity tensor?

4. (a) What is an electric dipole? Explain the difference between permanent and induced electric dipoles. Give an example of each. **[5]**

(b) An electric dipole is oriented along the x axis. It consists of a charge $-Q$ located at $x = -a/2$ and a charge $+Q$ located at $x = a/2$.

Show that the electric potential at a distance x along the x axis is given by

$$V(x) = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{x-a/2} - \frac{1}{x+a/2}\right\}.$$

[3]

(c) Show that for large distances $x \gg a$ the potential may be approximated by

$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^2}$$

where p is the electric dipole moment. **[3]**

(d) Given that the electric field on the axis is in the x direction, use the previous result to obtain an expression for the electric field on the axis. **[3]**

(e) A positive charge $+Q$ and a negative charge $-Q$ are placed in an electric field E. The force on the charges, due to this field, is balanced by the electrostatic attraction of the charges. Show that the electric dipole moment of the assembly is given by

$$p = \frac{Q^{3/2}}{(4\pi\varepsilon_0 E)^{1/2}}.$$

[3]

(f) Discuss how realistic this picture is for the induced electric polarisation of matter. **[3]**

5. A magnetic field wave propagating with wave vector $\mathbf{k}$ and angular frequency ω may be described by

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0 e^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r}-\omega t)} .$$

(a) By writing $\mathbf{B}$, $\mathbf{k}$ and $\mathbf{r}$ in terms of their rectangular components, or otherwise, show that

$$\partial\mathbf{B}/\partial t = -\mathrm{i}\omega\mathbf{B}, \quad \operatorname{div}\mathbf{B} = \mathrm{i}\mathbf{k}\cdot\mathbf{B} \quad \text{and} \quad \operatorname{curl}\mathbf{B} = \mathrm{i}\mathbf{k}\times\mathbf{B} .$$ **[6]**

(b) Use this result and the corresponding expressions for the electric field to show that for such wave-like fields the Maxwell equations in empty space:

$$\operatorname{div}\mathbf{E} = 0 \qquad \operatorname{curl}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$
$$\operatorname{div}\mathbf{B} = 0 \qquad \operatorname{curl}\mathbf{B} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

may be expressed as

$$\mathbf{k}\cdot\mathbf{E} = 0 \qquad \mathbf{k}\times\mathbf{E} = \omega\mathbf{B}$$
$$\mathbf{k}\cdot\mathbf{B} = 0 \qquad \mathbf{k}\times\mathbf{B} = -\frac{1}{c^2}\omega\mathbf{E} .$$ **[4]**

(c) Show that the electric field, the magnetic field and the direction of propagation are mutually perpendicular. **[5]**

(d) The Poynting vector $\mathbf{S}$ is defined (in free space) to be

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} .$$

Show that $\mathbf{S}$ points along the propagation of the electromagnetic wave.

What physical property does the Poynting vector represent? **[5]**

END