UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2005

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420D: ELECTROMAGNETISM

PH2420D R: ELECTROMAGNETISM - PAPER FOR RESIT CANDIDATES

MT324: ELECTROMAGNETIC THEORY

Time Allowed: **TWO** hours

Answer ***QUESTION ONE*** *and* ***TWO*** *other questions*

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators or CASIO fx85MS Calculators are permitted

PH2420D/46

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GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$\mathrm{m\ F^{-1}}$
Speed of light in vacuum	c	=	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$\mathrm{C\ kg^{-1}}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$\mathrm{W\ m^{-2}\ K^{-4}}$
Gas constant	R	=	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gravitational constant	G	=	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Acceleration due to gravity	g	=	9.81	$\mathrm{m\ s^{-2}}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.01×10^{5}	$\mathrm{N\ m^{-2}}$

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) Calculate the ratio of the electrostatic force to the gravitational force between a proton and an electron. How does this ratio depend on separation? **[4]**

(b) Explain the meaning of the statement that an electric field is conservative. When is an electric field not conservative? **[4]**

(c) Write down the expression for the force on a particle of charge q which is moving with velocity **v** through an electric field **E** and a magnetic field **B**. Hence show that if **E**, **B** and **v** are mutually perpendicular then it is possible for particles to move with a constant velocity in a straight line. What determines the magnitude of this velocity? **[4]**

(d) Show carefully that, in the absence of any surface charge, the normal component of **D** and the tangential component of **E** are continuous at the boundary between free space and a dielectric material. **[4]**

(e) Describe and explain qualitatively a possible mechanism for the finite electrical conductivity of metals. **[4]**

(f) Assuming the Poisson equation $\nabla^2 V = -\rho/\varepsilon_0$, show that the electric field in a cavity within a conductor is zero. **[4]**

PART MARKS

2. (a) State Gauss's law relating the flux of an electric field through a closed surface to the total electric charge enclosed. **[3]**

(b) A sphere of radius a contains a uniform charge Q distributed uniformly throughout its volume. How much charge is contained in a concentric sphere of radius r (where $r < a$)? **[3]**

(c) Using this result and Gauss's law, show that the magnitude of the electric field within the sphere varies with radius as

$$\frac{Qr}{4\pi\varepsilon_0 a^3}.$$ **[4]**

(d) What is the direction of the electric field? **[2]**

(e) The electron cloud of an atom may be modelled as a uniform distribution of negative charge Q within a sphere of radius a.

The atom is *polarised* if the centre of the electron cloud is displaced from the centre of the nucleus.

Using the result of (c), what is the force between the electron cloud and the nucleus of a polarised atom? **[3]**

(f) An applied electric field **E** will polarise the atom by pushing the electron cloud in one direction and the nucleus in the other. By balancing this force with the electrostatic attraction between electron cloud and nucleus, show that the induced dipole moment is given by

$$\mathbf{p} = 4\pi\varepsilon_0 a^3\mathbf{E}\ .$$ **[3]**

(g) Comment on the linearity of this expression. **[2]**

3. (a) The Maxwell equations in free space are:

$$\operatorname{div}\mathbf{E} = \rho/\varepsilon_0 \qquad \operatorname{curl}\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$\operatorname{div}\mathbf{B} = 0 \qquad \operatorname{curl}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

where the symbols have their usual meaning. Explain what is meant by the displacement current and identify it in the above equations. **[5]**

(b) By considering Ampere's law and the magnetic field in the vicinity of a charging capacitor, show that there is a need to introduce the displacement current. **[5]**

(c) Show that the displacement current is essential for the derivation of the propagation of electromagnetic waves by demonstrating that, in free space, **E** obeys the wave equation:

$$\nabla^2\mathbf{E} - \mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} = 0\,.$$ **[5]**

(d) A capacitor made from two parallel circular plates of radius a and separation d is charged by a current I. Calculate the magnetic field midway between the plates at a radial distance r ($r << a$) from the centre while the capacitor is being charged. **[5]**

Useful information:

$$\operatorname{curl}\operatorname{curl}\mathbf{E} = \operatorname{grad}\operatorname{div}\mathbf{E} - \nabla^2\mathbf{E}\,.$$

PART MARKS

4. The magnetic field at a point **r** away from an element d**l** of a wire carrying a current I is given by

$$\mathrm{d}\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\mathbf{l}\times\mathbf{r}}{r^3}.$$

(a) What is the direction of **B** for a long straight wire? Sketch this. **[4]**

(b) Using the relations at the end of this question, show that **B** may be written as the curl of the vector potential **A**, where

$$\mathrm{d}\mathbf{A} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\mathbf{l}}{r}.$$

[10]

(c) What is the direction of **A** for a long straight wire? Sketch this. **[2]**

(d) Find the magnetic field corresponding to the vector potential given by $\mathbf{A} = \frac{1}{2}(\mathbf{K}\times\mathbf{r})$ where **K** is a constant vector in the x direction and **r** is the position vector. **[4]**

Useful formulae:

$$\mathrm{grad}\left(\frac{1}{r}\right) = -\frac{\mathbf{r}}{r^3}$$

$$\mathrm{curl}\left(a\,\mathbf{b}\right) = a\,\mathrm{curl}\,\mathbf{b} - \mathbf{b}\times\mathrm{grad}\,a$$

5. (a) Define the magnetisation **M** and the magnetic susceptibility χ of a magnetic material and state a relationship between **B**, **H** and **M**. **[3]**

(b) Discuss briefly what is meant by *diamagnetic*, *paramagnetic* and *ferromagnetic* as applied to the magnetic properties of materials. **[6]**

(c) Sketch a B–H curve for a ferromagnetic material and hence explain why such materials can be used as permanent magnets. Why is the area enclosed by the curve of relevance to the design of transformers? **[6]**

(d) An iron compass needle of length 10 mm and cross section 1.0 mm^2 produces a B field of 0.01 T at the surface of its poles. Why is the magnetisation of the needle related to the B field by $\mathbf{M} = \mathbf{B}/\mu_0$? **[2]**

(e) The compass needle is placed at an angle of 30° to the earth's magnetic field of 4×10^{-5} T. Calculate the torque acting on the needle. **[3]**

END