UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2004

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420D: ELECTROMAGNETISM

PH2420D R: ELECTROMAGNETISM - PAPER FOR RESIT CANDIDATES

Time Allowed: **TWO** hours

*Answer **QUESTION ONE** and **TWO** other questions*

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators or CASIO fx85MS Calculators are permitted

PH2420D/44

2003-04

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^{2} kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\qquad$ $\pi \cong 3.142$ $\qquad$ $\log_e 10 \cong 2.303$

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) Equal charges Q are placed at *three* corners of a square of side a. What is the magnitude and direction of the electric field at the empty corner? **[4]**

(b) A bicycle wheel of radius 0.3 m. is spinning at a rate of one revolution per second. The bicycle is at rest. What is the value of the emf induced in the spokes of the wheel? You may assume the normal horizontal component of the earth's magnetic field is 10^{-4} T. **[4]**

(c) Explain (briefly) why placing a dielectric between the plates of a capacitor increases its capacitance. **[4]**

(d) Sketch the $B - H$ curve of a magnetic material. Describe briefly how the curve may be measured. **[4]**

(e) In a given rectangular Cartesian coordinate system the conductivity tensor of a two dimensional anisotropic substance is given by **[4]**

$$\sigma = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix}.$$

What is special about this coordinate system?

What is the magnitude and direction of the current density in the presence of an electric field represented by the vector

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}?$$

(f) Sketch the magnetic field lines in the vicinity of a magnetic dipole. What can you say about the flux of **B** through a closed surface surrounding the dipole? **[4]**

PART MARKS

2. (a) State Gauss's law relating the electric flux through a closed surface to the enclosed electric charge. **[2]**

(b) A parallel-plate capacitor caries a charge of $+Q$ on one plate and $-Q$ on the other. Show that the electric field within the capacitor has a magnitude $E = Q/\varepsilon_0 A$, where A is the area of the plates. State clearly any assumptions you make. In what direction does the **E** field point? **[4]**

(c) Show that the work δW done in moving a small charge δQ ($<< Q$) from one plate to the other is given by

$$\delta W = \frac{Q}{C}\delta Q$$

where C is the capacity of the capacitor. **[3]**

(d) Hence show that the work W done in charging the capacitor from zero to a final charge Q is given by

$$W = \frac{Q^2}{2C}\,.$$ **[3]**

(e) Explain how this result leads to the identification of an energy density U associated with an electric field E:

$$U = \frac{1}{2}\varepsilon_0 E^2\,.$$ **[3]**

(f) If the electron is modelled as a charged conducting sphere of radius a then show that the energy associated with its electric field is given by

$$\frac{e^2}{8\pi\varepsilon_0 a}\,.$$ **[3]**

(g) Discuss how this result might be related to the mass of the electron. **[2]**

3. (a) Define electric current density **J** and electrical conductivity σ. **[2]**

(b) By considering a cylinder of resistive material of length L, cross section area A and resistance R show that the conventional statement of Ohm's law: $I = V/R$ is equivalent to the expression $\mathbf{J} = \sigma\mathbf{E}$. **[4]**

(c) The mean velocity v of charges q and mass m moving in a resistive material, under the influence of an electric field, may be described by the equation

$$m\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{m}{\tau}v=qE\,.$$

The parameter τ is known as the *relaxation time*.

Give a justification of this equation. Include an explanation of the physical mechanisms that determine the relaxation time. **[6]**

(d) Show that the static conductivity of this medium may be expressed as

$$\sigma=\frac{Nq^2\tau}{m}$$

where N is the number of charges per unit volume. **[5]**

(e) If the material were subjected to an *alternating* electric field, outline how the above model may be used to obtain an expression for the a.c. conductivity. **[3]**

PART MARKS

4. (a) What is *displacement current*? **[2]**

(b) Show that the vector calculus identity div curl $\equiv 0$ implies that Ampère's law in free space

$$\mathrm{curl}\,\mathbf{B} = \mu_0 \mathbf{J}$$

and the equation of continuity

$$\mathrm{div}\,\mathbf{J} = -\frac{\partial \rho}{\partial t}$$

are incompatible. **[3]**

(c) By modifying Ampère's law by adding a further term **K**:

$$\mathrm{curl}\,\mathbf{B} = \mu_0 \mathbf{J} + \mathbf{K}$$

show that the incompatibility is removed if **K** satisfies

$$\mathrm{div}\,\mathbf{K} = \mu_0 \frac{\partial \rho}{\partial t}\,.$$ **[5]**

(d) Use Gauss's law to show that the modified Ampère's law expression may be written as

$$\mathrm{curl}\,\mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ **[5]**

(e) Show that the new term added to Ampère's law is equivalent to an effective electric current proportional to the rate of change of electric flux. Discuss the parallels/similarities with Faraday's law of electromagnetic induction. **[5]**

5. (a) The force between two charges and the force between two current elements in free space is given by

$$\mathbf{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2}\hat{\mathbf{r}}_{12}, \qquad \mathrm{d}\mathbf{F}_1 = \frac{\mu_0}{4\pi}\frac{I_1 I_2 \mathrm{d}\mathbf{l}_1 \times \left(\mathrm{d}\mathbf{l}_2 \times \hat{\mathbf{r}}_{12}\right)}{r_{12}^2}$$

where the symbols have their usual meaning.

Describe the sequence of arguments that lead from these expressions to the Maxwell equations:

$$\operatorname{div}\mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \operatorname{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\operatorname{div}\mathbf{B} = 0 \qquad \operatorname{curl}\mathbf{B} = \mu_0 \mathbf{J} + \mu_0\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Include a discussion of the definition of the electric and magnetic fields. **[15]**

(b) Show that the Maxwell equations imply the propagation of electromagnetic waves by demonstrating, for example, that in free space **E** obeys the wave equation

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0.$$ **[5]**

You may use the vector calculus identity

$$\text{curl curl} = \text{grad div} - \nabla^2 .$$

END