UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2003

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420D: ELECTROMAGNETISM

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators are permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$H\ m^{-1}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$F\ m^{-1}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$m\ F^{-1}$
Speed of light in vacuum	c	=	3.00×10^{8}	$m\ s^{-1}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$C\ kg^{-1}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\eta = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$J\ K^{-1}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$W\ m^{-2}\ K^{-4}$
Gas constant	R	=	8.31	$J\ mol^{-1}\ K^{-1}$
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	$N\ m^{2}\ kg^{-2}$
Acceleration due to gravity	g	=	9.81	$m\ s^{-2}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	$N\ m^{-2}$

MATHEMATICAL CONSTANTS

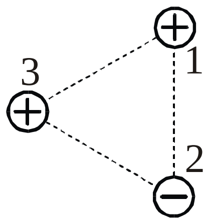
$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) The figure shows three charges of equal *magnitude*. Charges 1 and 3 are positive and charge 2 is negative.



Draw this figure and indicate on it arrows representing the magnitude and direction of the forces acting on charge 3 due to charge 1 and charge 2. **[2]**

Now draw a third arrow representing the magnitude and direction of the resultant force on charge 3. **[2]**

(b) The magnetic vector potential **A** is related to the magnetic field **B** through the integral relation

$$\iint_{\text{surface}} \mathbf{B}.\mathrm{d}\mathbf{a} = \oint_{\substack{\text{perimeter}\\ \text{of surface}}} \mathbf{A}.\mathrm{d}\mathbf{l}\,.$$

Express this relation in the differential curl form. **[2]**

How does the magnetic flux penetrating an area depend on the value of **A** at the area's perimeter? **[2]**

(c) Give a brief explanation of why Maxwell felt it necessary to introduce the displacement current into electromagnetism. **[4]**

(d) Write down the expression for the electric current density in terms of the conductivity tensor and the electric field. **[1]**

A two-dimensional anisotropic medium has a conductivity tensor given by $\begin{pmatrix} 0 & k \\ -k & 0 \end{pmatrix}$. What is the current density if the electric field points in the x direction? In this case what is the power dissipation in the medium? **[3]**

(e) In many cases the electric field is said to be a *conservative* field. What does this mean? When is the field *not* conservative? **[4]**

(f) State Kirchhoff's current and voltage laws. What physical principles do they embody? **[4]**

2. (a) State *Gauss's law* relating the flux of an electric field to electric charge. **[3]**

(b) Define *electric potential* and show that the potential V at a distance r from a point charge Q is given by

$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$ **[3]**

(c) A sphere of radius R contains charge Q. The charge density is uniform throughout the volume. How much charge is contained in a smaller concentric sphere of radius a? **[2]**

(d) Using this result together with Gauss's law, how does the electric field vary with radius within the sphere? **[3]**

(e) How does the electric field E vary with radius outside the sphere? **[3]**

(f) Sketch the variation of E with distance from the centre of the sphere. **[3]**

(g) A point charge $-Q$ is now placed at the centre of the sphere. Discuss the variation of E outside the sphere. **[3]**

3. (a) Two pith balls, each of mass m are suspended from a point by threads of length l. Each ball carries a charge Q and the resultant repulsion forces them a distance r apart. By balancing the gravitational force, Coulomb force and the tension in the threads show that the Coulomb force is related to the separation r by

$$F_C = \frac{mgr}{\sqrt{4l^2 - r^2}}.$$ **[7]**

(b) In the limit that the separation r is very much less than the length l, show that r is related to the charge Q by

$$r^3 = \frac{2lQ^2}{4\pi\varepsilon_0 mg}.$$ **[4]**

(c) Calculate the separation of the balls in the case where the threads are 10 cm long, the balls each weigh 1 mg and they both carry one electron of excess charge. (Use the $r \ll l$ limit.) **[4]**

(d) Discuss the feasibility of using such a measurement to determine the electronic charge. You should include a consideration of the size of the balls. **[5]**

PART MARKS

4. (a) State *Faraday's law* of electromagnetic induction. **[3]**

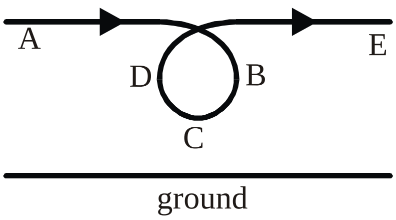
(b) An aeroplane is travelling with velocity v in a horizontal direction. The vertical component of the earth's magnetic field is given by B_z. If the wing span of the aeroplane is of length l show that Faraday's law implies there will be a voltage V induced between the wing tips of magnitude

$$V = B_z l v.$$ **[4]**

(c) A Boeing 747 aircraft has a wing span of 60 m and a cruising speed of 250 ms^{-1}. What is the magnitude of the voltage induced between the wing tips? You may take the vertical component of the earth's field to be 6.5×10^{-5} T. **[2]**

(d) In order to measure this effect a student places a voltmeter in the centre of the aircraft and connects wires from the meter to the tip of each wing. No voltage is observed. Why? **[5]**

(e) The voltmeter is now connected to a horizontal wire loop. Still no deflection of the meter is observed. However when the aeroplane executes a somersault as shown in the figure, then a voltage is indicated on the meter.



Assuming the earth's field is vertical, describe and explain the voltage observed during the trajectory. You may find it useful to refer to the points A, B, C, D and E.

What difference would be observed if the earth's field were directed at an angle of 30 degrees tot he vertical downward direction, with its horizontal component in the direction of flight? **[6]**

PART MARKS

5. An electric field wave propagating in the **k** direction may be written as

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{\mathrm{i}(\mathbf{k}.\mathbf{r}-\omega t)} .$$

(a) By writing **E**, **k** and **r** in terms of their Cartesian components (or otherwise), show that

$$\text{div }\mathbf{E} = \mathrm{i}\mathbf{k}.\mathbf{E} \quad \text{and} \quad \text{curl }\mathbf{E} = \mathrm{i}\mathbf{k}\times\mathbf{E} .$$ **[7]**

(b) Use this result (and the corresponding result for the **B** field) to show that for harmonic fields the *in vacuo* Maxwell equations

$$\text{div }\mathbf{E} = 0 \qquad \text{curl }\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\text{div }\mathbf{B} = 0 \qquad \text{curl }\mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

may be expressed as

$$\mathbf{k}.\mathbf{E} = 0 \qquad \mathbf{k}\times\mathbf{E} = \omega\mathbf{B}$$

$$\mathbf{k}.\mathbf{B} = 0 \qquad \mathbf{k}\times\mathbf{B} = -\frac{1}{c^2}\omega\mathbf{E} .$$ **[3]**

(c) Show that the electric field, the magnetic field and the direction of propagation are mutually perpendicular. **[5]**

(d) Show that the Poynting vector $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$ is parellel to the direction of propagation.. What does this vector represent?

You may use the results:

$$\text{curl curl} = \text{grad div} - \nabla^2$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{b}(\mathbf{a}\cdot\mathbf{c}) - \mathbf{c}(\mathbf{a}\cdot\mathbf{b}) .$$ **[5]**