UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2002

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420C: ELECTROMAGNETISM

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators are permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$\mathrm{m\ F^{-1}}$
Speed of light in vacuum	c	=	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$\mathrm{C\ kg^{-1}}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$\mathrm{W\ m^{-2}\ K^{-4}}$
Gas constant	R	=	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gravitational constant	G	=	6.67×10^{-11}	$\mathrm{N\ m^2\ kg^{-2}}$
Acceleration due to gravity	g	=	9.81	$\mathrm{m\ s^{-2}}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	$\mathrm{m^3}$
One standard atmosphere	P_0	=	1.01×10^{5}	$\mathrm{N\ m^{-2}}$

MATHEMATICAL CONSTANTS

$e \cong 2.718 \qquad \pi \cong 3.142 \qquad \log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) How is the ampère defined? Outline how electric currents may be standardised using the magnetic forces they produce. **[4]**

   (b) At a boundary between a dielectric and free space show that the normal component of **D** and the tangential component of **E** are continuous. **[4]**

   (c) Assuming the Poisson equation $\nabla^2 V = -\rho/\varepsilon_0$, show that the electric field in an empty cavity within a conductor is zero. No current is flowing in he conductor. **[4]**

   (d) What is meant by *self inductance* and *mutual inductance*? Explain briefly the connection between these phenomena and the working of a transformer. **[4]**

   (e) What is meant by a gauge transformation in electromagnetism? **[4]**

   (f) Explain how the continuity equation $\mathrm{div}\mathbf{j} = -\partial\rho/\partial t$ relates to the law of conservation of charge. Include a mention of global and local conservation. **[4]**

PART MARKS

2. (a) Faraday's law of electromagnetic induction may be written

$$\varepsilon = -\frac{d\Phi}{dt}.$$

Explain what this equation means. **[3]**

(b) In a ballistic galvanometer the coil assembly has significant inertia so that for a short pulse of current the deflection is proportional to the charge passed through the coil. This can form the basis of a system for measuring magnetic field B. A small wire loop of area a is connected to the galvanometer. The loop is suddenly moved out of the magnetic field. Using Faraday's law of induction show that the deflection of the galvanometer will be proportional to Ba/R where R is the resistance of the circuit. **[6]**

(c) The loop is oriented perpendicular to the field, and moved so that the field it experiences reduces uniformly to zero in a time t. Show that the minimum work done in the process is given by **[3]**

$$W = \frac{B^2a^2}{Rt}.$$

(d) Calculate the work done when a loop of area 10^{-4} m^2 is withdrawn from a field of 1 T in a time of 1 s. You may assume the resistance of the circuit is 1 Ω. **[4]**

(e) Since work is done during the process, this implies the coil experiences a force as it moves. Discuss the origin of this force. **[4]**

3. (a) From the point of view of electromagnetism, matter may be regarded as an assembly of charges and currents. Explain the distinction between free and bound charges and currents. **[3]**

(b) The bound charge density may be expressed as **[4]**

$$\rho_{\text{bound}} = -\text{div}\mathbf{P} \,.$$

Comment on this relation and discuss the microscopic interpretation of the polarisation vector **P**.

(c) Show how the microscopic time-independent field equation

$$\text{div}\mathbf{E} = \left(\rho_{\text{free}} + \rho_{\text{bound}}\right)/\varepsilon_0$$

may be re-expressed, by the introduction of the polarisation **P**, as a *macroscopic* field equation in terms of the field vector **D**. **[5]**

Write down the expression for this macroscopic field vector **D** in terms of the vectors **E** and **P**.

(d) Why are the equations for div**B** and curl**E** *not* modified in the presence of dielectric and magnetic media? **[4]**

(e) Explain the distinction between *microscopic* and *macroscopic* fields. **[4]**

4. (a) The Maxwell equations in free space may be written as

$$\mathrm{div}\mathbf{E} = \rho/\varepsilon_0 \qquad \mathrm{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\mathrm{div}\mathbf{B} = 0 \qquad \mathrm{curl}\mathbf{B} = \mu_0\varepsilon_0\frac{\partial \mathbf{E}}{\partial t} + \mu_0\mathbf{j}$$

where the symbols have their usual meaning.

Which parts of the equations embodies:

i) Gauss's law, **[8]**

ii) Ampère's law,

iii) Faraday's law of electromagnetic induction,

iv) the displacement current?

(b) Show that the Maxwell equations imply the propagation of electromagnetic waves by demonstrating, for example, that in free space **E** obeys the wave equation

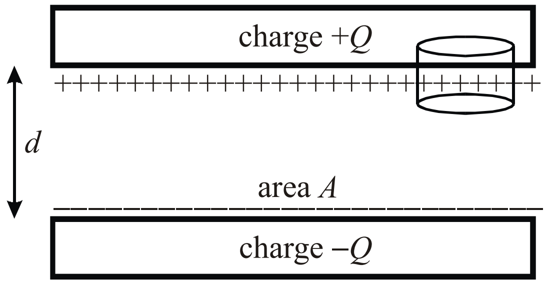
$$\nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}.$$ **[5]**

(c) Obtain the speed of propagation of these waves in terms of the free space constants μ_0 and ε_0. **[2]**

(d) An electric field wave propagating in the **k** direction may be written as $\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{\mathrm{i}(\mathbf{k}.\mathbf{r}-\omega t)}$. For such a wave you may assume that $\mathrm{div}\mathbf{E} = \mathrm{i}\mathbf{k}.\mathbf{E}$ and $\mathrm{curl}\mathbf{E} = \mathrm{i}\mathbf{k}\times\mathbf{E}$. Using this result and the corresponding relations for the **B** field, show that the **E** field, the **B** field and the direction of propagation are mutually perpendicular. **[5]**

Hint: you may find the identity $\mathrm{curl\ curl} = \mathrm{grad\ div} - \nabla^2$ useful.

5. The figure shows a parallel plate capacitor. The plates each have an area A and they are separated by a distance d. The top plate carries a charge $+Q$ and the bottom plate carries charge $-Q$, uniformly distributed.



(a) By referring to the Gaussian pill box indicated in the figure, show that the electric field within the capacitor is uniform and that it has a magnitude $E = Q/\varepsilon_0 A$. In what direction does the **E** field point? **[4]**

(b) Show that the work done in moving a small charge q ($\ll Q$) from the bottom to the top plate is given by **[4]**

$$W = \frac{qQ}{C}$$

Where C is the capacitance of the capacitor.

(c) Hence show that the work done in charging the capacitor from zero to a charge Q is **[3]**

$$W = \frac{Q^2}{2C}.$$

(d) Explain how this result leads to the identification of an energy density U_E of the electric field **[3]**

$$U_E = \frac{1}{2}\varepsilon_0 E^2.$$

Discuss the assumptions made in this identification.

(e) A parallel plate capacitor is constructed from two square metal sheets 10^{-1} m × 10^{-1} m and spaced apart by 10^{-3} m. The capacitor is charged to a potential difference of 10^3 V. **[3]**

Calculate the electric field between the plates?

(f) Calculate the energy contained in the electric field? **[3]**