UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2001

For Internal Students of

Royal Holloway

<u>DO NOT TURN OVER UNTIL TOLD TO BEGIN</u>

PH2420C: ELECTROMAGNETISM

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Only CASIO fx85WA Calculators ARE permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	$\mathrm{H\ m^{-1}}$
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	$\mathrm{m\ F^{-1}}$
Speed of light in vacuum	c	=	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	$\mathrm{C\ kg^{-1}}$
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	$\mathrm{J\ K^{-1}}$
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	$\mathrm{W\ m^{-2}\ K^{-4}}$
Gas constant	R	=	8.31	$\mathrm{J\ mol^{-1}\ K^{-1}}$
Avogadro constant	N_A	=	6.02×10^{23}	$\mathrm{mol^{-1}}$
Gravitational constant	G	=	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Acceleration due to gravity	g	=	9.81	$\mathrm{m\ s^{-2}}$
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	$\mathrm{m^{3}}$
One standard atmosphere	P_0	=	1.01×10^{5}	$\mathrm{N\ m^{-2}}$

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\quad \pi \cong 3.142$ $\quad \log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) A square of side a has a charge $+Q$ placed at each corner. Calculate the electric potential V at the centre of the square. Where is the zero of electric potential? What is the electric field at the centre of the square? Explain your answer. **[4]**

(b) Show that the tangential component of the electric field **E** and the normal component of the field **D** are continuous at the boundary between a dielectric and free space. **[4]**

(c) Describe and explain qualitatively the mechanism for the electrical resistivity of metallic conductors. **[4]**

(d) Show how the equations $\mathrm{curl}\mathbf{E} = -\partial\mathbf{B}/\partial t$ and $\mathrm{curl}\mathbf{B} = \mu_0\varepsilon_0\,\partial\mathbf{E}/\partial t$ (where the symbols have their conventional meanings) lead to a wave equation for the electric field **E**. What are these waves and what is their speed? [You may use the identity $\mathrm{curl}\,\mathrm{curl} = \mathrm{grad}\,\mathrm{div} - \nabla^2$] **[4]**

(e) Write down the relation between electric field and electric current density for a linear, *an*isotropic, homogeneous conductor. Explain, briefly, the meaning of the term *tensor*. **[4]**

(f) From the relation **B** = curl**A**, where the symbols have their conventional meanings, deduce the relation between the flux contained within a closed loop and the line integral of **A** round that loop. **[4]**

2. (a) Ampère's law, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$ relates the line integral of the magnetic field $\mathbf{B}$ around a closed loop to the current I passing through the loop, in free space.

Show that this law may be expressed in terms of the curl of $\mathbf{B}$ as $\text{curl}\,\mathbf{B} = \mu_0 \mathbf{J}$ where $\mathbf{J}$ is the current density. **[3]**

(b) Demonstrate that there is an inconsistency in Ampère's law when considering the space between the plates of a capacitor. **[5]**

(c) The charge on a parallel plate capacitor (in a vacuum) may be written as

$$Q = \varepsilon_0 A E$$

where E is the electric field and A is the area of the plates. Use this relation to show that the inconsistency referred to in (b) above may be removed through the introduction of a fictitious current i given by

$$i = \varepsilon_0 A \frac{\partial E}{\partial t}.$$ **[5]**

(d) Why is i called the *displacement current*? **[3]**

(e) Show that including the effect of the displacement current, the curl of $\mathbf{B}$ may be written as

$$\text{curl}\,\mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$ **[4]**

3. (a) Write down the formula for the force exerted on a charged particle moving in an electromagnetic field. **[3]**

(b) With the help of the above expression show that the potential difference developed across the ends of a straight conductor of vector length $\mathbf{l}$ in a magnetic field $\mathbf{B}$ and moving with velocity $\mathbf{v}$ is given by $V = \mathbf{B} \cdot \mathbf{l} \times \mathbf{v}$. **[6]**

(c) By use of a suitable diagram, show that this result leads to Faraday's law of electromagnetic induction $V = -\partial \Phi / \partial t$. **[7]**

(d) An aeroplane has a wingspan of 10m. What potential difference would be generated between the wing tips when it is travelling at a speed of 100 ms^{-1}? Assume the vertical component of the earth's magnetic field is 10^{-4} T. Discuss briefly whether this is measurable. **[4]**

PART MARKS

4. The magnetic field at a point **r** away from an element d**l** of a wire carrying a current I is given by

$$\mathrm{d}\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}\mathbf{l} \times \mathbf{r}}{r^3} .$$

(a) What is the direction of **B** for a long straight wire? Show this on a suitable sketch. **[4]**

(b) Using the relations given below, show that **B** may be written as the curl of the vector potential **A**, where **[7]**

$$\mathrm{d}\mathbf{A} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}\mathbf{l}}{r} .$$

(c) What is the direction of **A** for a long straight wire? Show this on a suitable sketch. **[4]**

(d) A long straight wire carries a current of 1 ampère. What is the magnitude of **B** and **A** a distance 1m away from the wire? **[5]**

Useful relations:

$$\frac{\mathbf{r}}{r^3} = -\mathrm{grad}\frac{1}{r}$$

$$\mathrm{curl}\left(a\mathbf{b}\right) = a\,\mathrm{curl}\,\mathbf{b} - \mathbf{b} \times \mathrm{grad}\,a .$$

5. (a) Assuming the Poisson equation $\nabla^2 V = -\rho/\varepsilon_0$, show that the electric field within a cavity in a conductor is zero. **[6]**

(b) Explain how this result may be used as the basis for a test of the *inverse square law* of the Coulomb force. **[5]**

(c) Describe an experiment to verify the Coulomb inverse square law to a very high accuracy. Pay particular attention to the methods whereby this high accuracy is obtained and errors minimised. **[6]**

(d) Discuss a possible explanation *if* it were to be discovered that the inverse square law were not quite correct. **[3]**