UNIVERSITY OF LONDON

BSc and MSci EXAMINATION 2000

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH2420B: ELECTROMAGNETISM

Time Allowed: TWO hours

Answer QUESTION ONE and TWO other questions

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Calculators ARE permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^3
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

PART MARKS

ANSWER **ONLY FIVE** sections of ***Question One***.

You are advised not to spend more than **40 minutes** answering ***Question One***.

1. (a) Describe and sketch the motion of an *initially stationary* charged particle in *parallel* electric and magnetic fields. **[4]**

(b) Show how the equations $\mathrm{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t$ and $\mathrm{curl}\,\mathbf{B} = \mu_0\varepsilon_0\,\partial\mathbf{E}/\partial t$ lead to the existence of electromagnetic waves. **[4]**
[You may use the identity $\mathrm{curl}\,\mathrm{curl} = \mathrm{grad}\,\mathrm{div} - \nabla^2$]

(c) What is meant by *self inductance* and *mutual inductance*? Explain briefly the relevance of these phenomena to the working of a transformer. **[4]**

(d) Explain how the continuity equation $\mathrm{div}\mathbf{J} = -\partial\rho/\partial t$ relates to the law of conservation of charge. **[4]**

(e) Ampère's law may be written $\oint \mathbf{B}.\mathrm{d}\mathbf{l} = \mu_0 I$. Explain clearly the meaning of this equation. Use this law to find the magnetic field at a distance R from a straight, infinitely long current-carrying wire. **[4]**

(f) Show how Kirchhoff's voltage law follows from the equation $\mathrm{curl}\mathbf{E} = 0$. Under what circumstances would Kirchhoff's voltage law fail? **[4]**

2. (a) State Gauss's law relating the flux of an electric field **E** out of a closed surface to the total electric charge enclosed. **[3]**

   (b) A spherical conducting shell of radius R contains charge Q distributed uniformly over its surface. Using Gauss's law or otherwise show that the electric field at the outer surface of the sphere has magnitude σ/ε_0 where σ is the areal charge density. And show that the field points normal to the surface. **[4]**

   (c) Show that the electric field at the inner surface of the sphere is zero. **[3]**

   (d) What is the magnitude of the electric field at a distance $r > R$ from the centre of the sphere? **[3]**

   (e) A point charge of $-Q$ is now placed at the centre of the sphere. How does the electric field vary inside the sphere and outside the sphere? Plot this. **[4]**

   (f) What is the electric dipole moment of this system? Explain your reasoning. **[3]**

3. (a) State Faraday's law of electromagnetic induction that relates electromotive force to the variation of magnetic flux. **[3]**

   (b) In a *ballistic galvanometer* the coil assembly has significant inertia so that for a short pulse of current the deflection is proportional to the charge passed through the coil. This can form the basis of a system for measuring magnetic field B. A wire loop of area a is connected to the galvanometer. The loop is suddenly moved out of the magnetic field. Using Faraday's law of electromagnetic induction show that the deflection of the galvanometer will be proportional to Ba/R where R is the resistance of the circuit. **[8]**

   (c) The loop is moved so that the field it experiences reduces uniformly to zero in a time Δt. Show that the minimum work done in the process is given by

$$W = \frac{B^2a^2}{R\Delta t}$$

[5]

   (d) Since work is done, this implies the coil is experiencing a force as it moves. Discuss the origin of this force. **[4]**

PART MARKS

4. (a) Define the term *capacitance*. What units are used to measure capacitance? **[3]**

(b) Show how does the capacitance of a parallel plate capacitor depends on the area A and the separation d of the plates? **[2]**

(c) A parallel plate capacitor in a vacuum has a charge Q. Show that this charge is related to the electric field between the plates by

$$Q = \varepsilon_0 A E \,.$$ **[3]**

(d) Show that an electric current I flowing into one plate and out of the other will cause a varying electric field given by

$$I = \varepsilon_0 A \frac{\partial E}{\partial t} \,.$$ **[3]**

(e) In terms of Ampère's law, which may be expressed as

$$\oint \mathbf{B}.\mathrm{d}\mathbf{l} = \mu_0 I$$

for the wires outside the capacitor, explain how a time-varying electric field will produce a magnetic field between the capacitor plates. **[5]**

(f) Explain the meaning of the term *displacement current*. **[4]**

PART MARKS

5. (a) Define electric current density **J** and electrical conductivity σ. **[2]**

(b) By considering a cylinder of resistive material of length L, cross section area A and resistance R, show that Ohm's law can be written as $\mathbf{J} = \sigma \mathbf{E}$. **[4]**

(c) The mean velocity v of particles with charge q and mass m moving under the influence of an electric field E in a resistive medium may be modelled by the equation

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{m}{\tau}v = qE$$

where τ is the relaxation time. Explain carefully the physical information contained in this equation. **[6]**

(d) Show that the d.c. conductivity of this medium may be expressed as

$$\sigma = \frac{Nq^2\tau}{m}$$

where N is the number of charges per unit volume. **[5]**

(e) For alternating currents the medium is conveniently described in terms of a *complex* conductivity. Explain, within the framework of the above model, why this is. **[3]**