Finding Detours is Fixed-parameter Tractable

Ivona Bezáková, Radu Curticapean, Holger Dell, Fedor V. Fomin

Gregory60

- Longest Path: Given a graph G and integer k, decide whether G contains a path of length at least k?

Longest Path: Fixed-parameter tractability

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- Win/Win: If the treewidth is large, there is a long path

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- Otherwise do DP

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- Determenistic: $2.597^k \cdot n^{O(1)}$ Zehavi [2013]

Longest Path of Gregory: 33 years

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MR738059 (85m:05048) 05C45 05C20
Gutin, G. M. [Gutin, Gregory] (2-GOME)
Criterion for complete bipartite digraphs to be Hamiltonian. (Russian. English summary)
Vestsī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk **1984**, *no. 1*, 109–110.

From the text (translated from the Russian): "A complete bipartite digraph $B = (V, W; A)$ has a set of vertices $X = V \cup W$, where V and W form a partition of the points of B and A is the set of arcs. By B^d we denote a digraph obtained from B after redirection of all its arcs. With every complete bipartite digraph $B = (V, W; A)$ we associate a bipartite nondirected graph $\mathrm{GR}(B)$. V and W form a partition of the points of $\mathrm{GR}(B)$ and the edge $\{v, w\}$ enters into $\mathrm{GR}(B)$ if and only if the arc $(v, w) \in A$ and $v \in V$, $w \in W$. Theorem: A necessary and sufficient condition for a complete bipartite digraph $B = (V, W; A)$ to have a Hamiltonian cycle is that B be strong and the graphs $\mathrm{GR}(B)$ and $\mathrm{GR}(B^d)$ have 1-factors."

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Gutin, G. M. [Gutin, Gregory] (2-GOME)
Cycles in strong n-partite tournaments. (Russian. English summary)
Vestsī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk **1984,** *no. 5,* 105–106.

It is proved that every strong n-partite tournament (i.e. a strongly directed complete n-partite graph) with $n\geq 4$, all of whose parts have at least two vertices, contains a cycle of length $n+1$ or $n+2$. However, for every integer $n\geq 2$ there exists a strong n-partite tournament with all parts of cardinality 2 containing no cycle of length $n+1$. Thus a problem posed by J. A. Bondy [J. London Math. Soc. (2) 14 (1976), no. 2, 277–282; MR0450115] is solved.

There are misprints on page 106. *J. Bosák*

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MR811155 (87b:05060) 05C20 05C45 68R10
Gutin, G. M. [Gutin, Gregory]
Effective characterization of complete bipartite digraphs having Hamiltonian paths. (Russian)
Kibernetika (Kiev) **1985,** *no. 4,* 124–125.

A bipartite directed graph D is said to be complete bipartite if each pair of vertices from different bipartition classes of D is joined by at least one edge. A directed almost-factor of D is a spanning subgraph F of D such that one of its connected components is a directed path while the others are directed circuits. A directed graph is said to be traceable if it has a directed Hamiltonian path.

The main theorem states that a complete bipartite directed graph D is traceable if and only if it has an almost-factor. Another theorem, obtained as a corollary of the main theorem, enables one, with the help of results of J. Hopcroft and R. Karp, to find an effective algorithm to decide whether a given graph has a directed Hamiltonian path. *B. Zelinka*

Longest Path of Gregory: 33 years

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MR3466628 Pending Barbero, Florian; Gutin, Gregory; Jones, Mark; Sheng, Bin; Yeo, Anders Linear-vertex kernel for the problem of packing r-stars into a graph without long induced paths. *Inform. Process. Lett.* 116 (2016), no. 6, 433–436. 94C15 (05C85 68Q25 68R10)
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UBB

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UBB

Another Longest Path of Gregory: Parameterization above guarantee

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This talk

Longest Path + Above guarantee parameterization

- **Longest Detour**: Given graph G, vertices s and t, and integer k. Is there an (s,t)- path in G of length at least $dist(s,t)+k$

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- Is the problem FPT parameterized by k?

- Longest Detour: Given graph G, vertices s and t, and integer k. Is there an (s,t)- path in G of length at least $dist(s,t)+k$
- Is the problem in P for fixed k?
- Is the problem FPT parameterized by k?
- THEOREM: Longest Detour is solvable in time $2^{O(k)}n^{O(1)}$

Win/Win?

- Chuzhoy (2015): If the treewidth of G is more than k^{19} poly(log k), then G contains $k \times k$-grid as a minor

Win/Win?

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- We can assume that G is 2-connected

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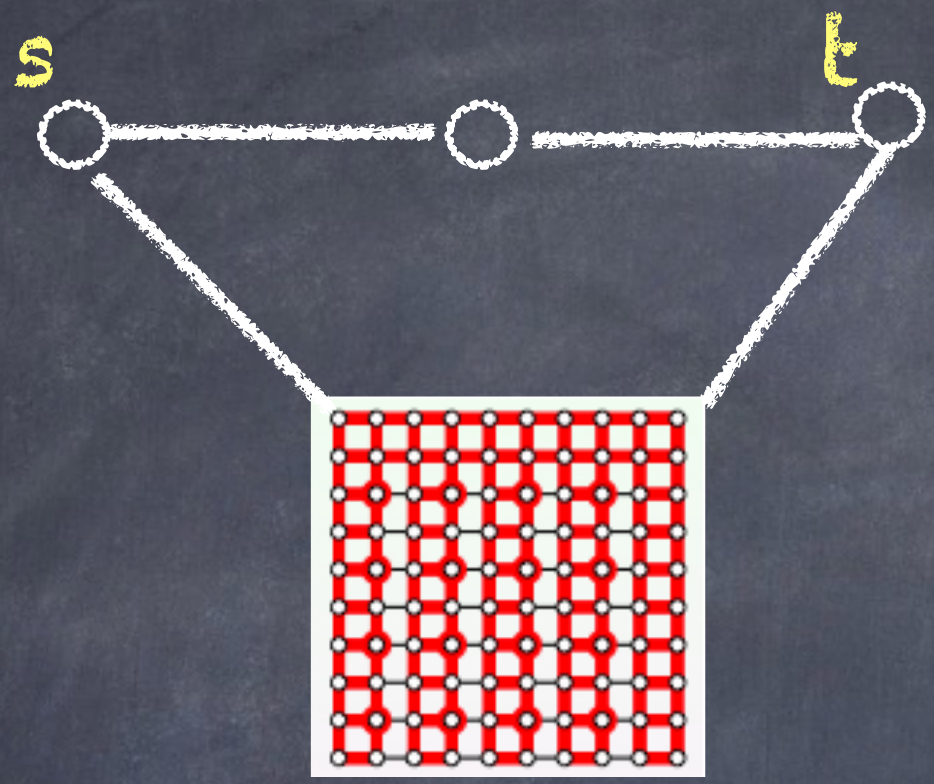
- If the treewidth of G is less than k^{19} use DP (time $2^{O(tw(G))}n$)

Win/Win?

- We can assume that G is 2-connected
- If the treewidth of G is less than k^{19} use DP (time $2^{O(tw(G))}n$)
- Otherwise use $k \times k$-grid for rerouting



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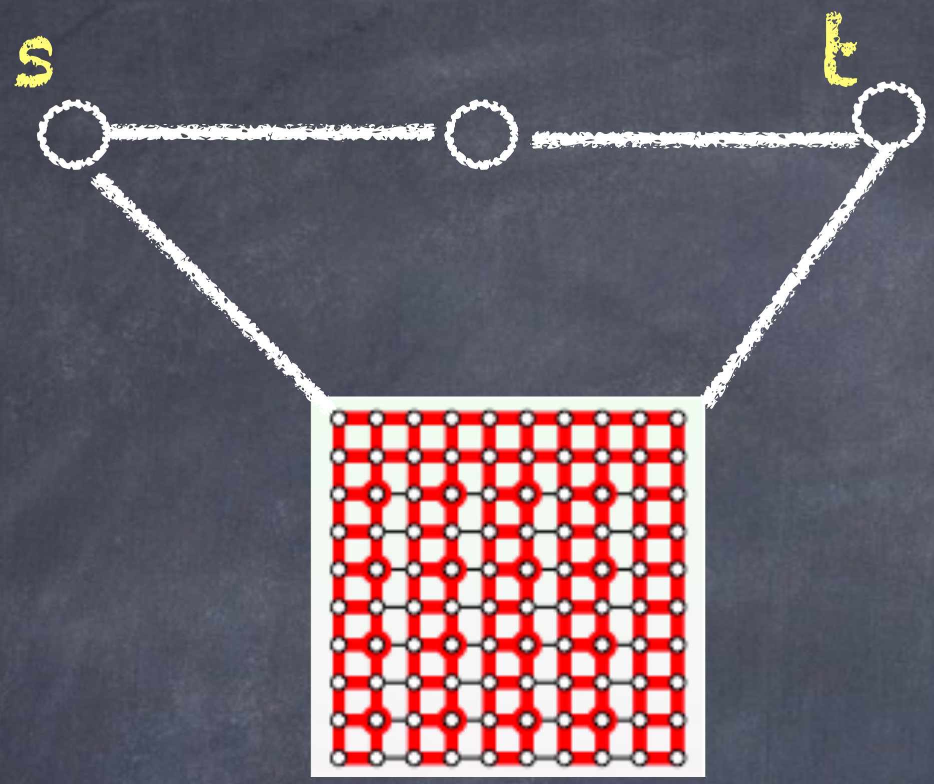
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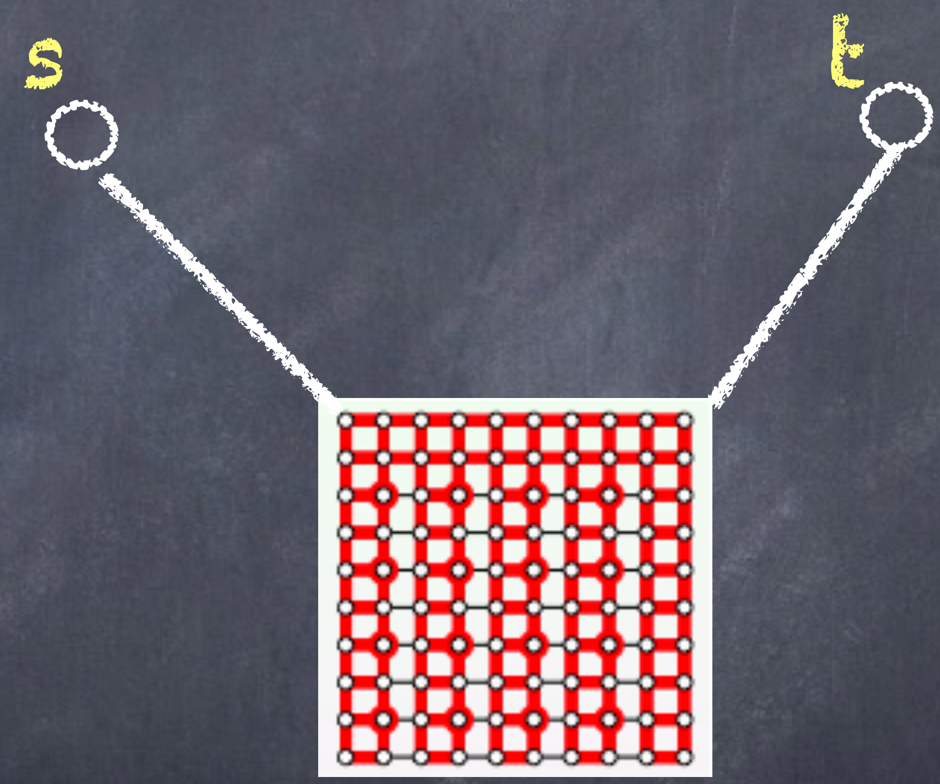
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Win/Win?

- This gives an algorithm solving Longest Detour in time $\exp(k^{19})\, n^{O(1)}$

Win/Win?

Win/Win?

- Can we exclude something simpler than a grid?

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- For example, if we exclude a k-cycle, the treewidth is $O(k)$.

Win/Win?

- Can we exclude something simpler than a grid?
- For example, if we exclude a k-cycle, the treewidth is $O(k)$.
- But k-cycle is not enough complicated for rerouting...

What graph

- Can be used for k-detour
- When excluded as a minor guarantees linear (in k) treewidth?

Combinatorial result

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- Graph F: Take K_4 and subdivide every edge k times

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- Graph F: Take K_4 and subdivide every edge k times
- F is the right graph!!!
  - Every F-minor-free graph has treewidth at most $32k$
  - Every (s,t)-shortest path in a graph containing F as a minor has a k-detour.

proof

proof

treewidth at least k is (approximately) equivalent of having a k-linked set

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Leaf and Seymour (2015): structure of k-linked sets

proof

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Leaf and Seymour (2015): structure of k-linked sets

Raymond and Thilikos (2016): Wheel excluding

What can be other "above guarantee" variants of Longest Path?

- Girth (FF, Lokshtanov, Saurabh, Zehavi)
- Degeneracy (FF, Golovach)

What about another passion of Gregory?

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- Longest Directed Detour: Given a digraph G, vertices s and t, and integer k. Is there an (s,t)- path in G of length at least $dist(s,t)+k$?

What about another passion of Gregory?

- Longest Directed Detour: Given a digraph G, vertices s and t, and integer k. Is there an (s,t)- path in G of length at least $dist(s,t)+k$?
- We do not know even if $poly(n,k)$ algorithm exist.

Remark

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- Exact Directed Detour: Given a digraph G, vertices s and t, and integer k. Is there an (s,t)- path in G of length exactly $dist(s,t)+k$?

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- Exact Directed Detour is FPT.

Happy Birthday, Gregory!!!