Three generations on 29th February

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Carrie Grant, from the BBC programme The One Show, talked with me recently about the Keogh family: they are in the Guinness Book of Records as the only recorded family with three consecutive generations all born on 29th February. She was interested to know how likely this was, and whether there could be other families out there that this has happened to. The programme went out on 29th February 2012.

This document gives a few extra details, and talks about some of the subtleties, that we didn't have time to include. I've not assumed any knowledge of statistics in what I write below.

How likely are you to be born on 29th February? A leap year happens only once every four years. The number of days in four years is $3 \times 365 + 366 = 1461$, as there are three years with the standard number of years, and one leap year with 366 days. So, assuming that all birthdates are equally likely, the chances you are born on 29th February is 1 in 1461. This is quite likely: there are over 60 million people in the UK, so you'd expect over 40 000 people with this birthday ($60\,000\,000/1461 = 41\,067.8$). Is the assumption that all birthdays are equally likely a good one? It's a good first approximation, but here's a link to an article that indicates that, in the U.S. at least, more babies are born in the late summer and early autumn than in other times of the year: `http://www.panix.com/~murphy/bday.html`. But let's assume that the 1 in 1461 chance we've calculated is accurate.

What about the chance that there are three generations born on 29th February? Let's start with a simpler problem. What are the chances that you, your mother and her mother are all born on 29th February? It seems reasonable to assume that the birthdays of your ancestors don't affect your birthday at all (using more technical terminology, I'm assuming that the three events are *independent*). The chance that all three of you are born on 29th February is one in $1461 \times 1461 \times 1461$, or one in 3 118 535 181 (one in 3.1 billion).

But you have four grandparents, and we're really interested in the chances that you, *any* grandparent, and their child/your parent are all born on 29th February. Because there are four grandparents, the chance is multiplied by about 4, giving a chance of about 4 in 3 118 535 181. Why do I say 'about' here? This is because multiplying by 4 overestimates the chance a little, as (for example) we've counted twice the (very unlikely) event that you, *two* of your grandparents and the people between in your family tree are all born on 29th February. Taking extra care of this doesn't affect the calculation much: the answer is still very close to 4 in 3.1 billion.

Are there likely to be any other families like the Keogh's out there, just unrecorded? The world population is currently about 7 billion, so you'd expect there to be about

$$7\,000\,000\,000 \times \frac{4}{3\,100\,000\,000} \approx 9$$

people in the world born on 29th February, sharing their birthday with a parent, and that parent's parent. That's how many you would expect, but how likely is it that there is no one else? One way of trying to estimate this (using an approximation by a Poisson distribution, under another simplifying independence assumption) would give a chance of 1 in 8100 (using the fact that $e^9 \approx 8100$). Insisting that the parent and grandparent are still around will improve the odds that the Keogh's are unique significantly: you would expect fewer than 9 people in the world with the right parent and grandparent still around, but without more data it's hard to put a figure on this.

How does a 4 in 3.1 billion chance compare with being struck by lightening? Say 30 people are struck by lightening each year in the UK (this is a low estimate, if you read the Tornado and Storm Research Organisation's webpage). With a life expectancy of about 80 years, the chance that someone will be struck by lightening in their life will be about 30×80 in 60 000 000 000, or about 1 in 25 000. So being struck by lightening is more than 30 thousand times more likely than being in the Keogh's position; the calculation I've done here is:

$$\frac{1/25\,000}{4/3\,100\,000\,000} = 31\,000.$$

What next? If you are interested in these kinds of problems, then the techniques of statistics will give you more information, and much greater understanding, than I give here. So (if you haven't already), studying some statistics is the way forward. And if you're interested in a birthday problem with some real mathematical depth, try searching for the 'birthday paradox'.

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