4.

Broken Odds and the Favourite-Longshot Bias in British On-Course Betting

4.1 – INTRODUCTION

In bookmaker markets bettors place bets at fixed-odds set by the bookmakers[1]. In practice, bookmakers post the odds they offer from a discrete list - this is what will be referred to as the *price-grid*[2]. In general, the intervals between 'quotable' odds (the *tick-size*) increases as the odds levels increase. In other words, the tick-size is narrower for favourites than for longshots. This chapter shows that this feature of the bookmaker price-grid can give rise to a 'built-in' favourite-longshot bias.

The second part of the chapter looks at pari-mutuel or tote markets - the main form of betting outside the UK, where typically a tote monopoly exists. In such markets, often a reverse favourite-longshot bias is observed: the expected returns from backing favourites are inferior to the expected returns from backing longshots. The chapter shows that this distinction from bookmaker markets may be attributed to differences in the price-grid. In pari-mutuel betting, stakes are all placed into a pool which is shared by the holders of winning tickets. The peculiar feature of these markets is that prior to distribution, the pool is first taxed by the authorities (the so

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[1] An alternative is to have the bet settled at the *starting odds*, which are the fixed-odds that will come to prevail at the start of the race.

[2] The actual price grid used by British bookmakers is discussed in Section 4.3 and presented in more detail Appendix 4.A.1: Table 4.A.1. The grid's existence and why it persists is also discussed in Section 4.3.

called *track-take*) and then payout rates (the *dividends*) are rounded down in a process called *breakage*. The track-take and breakage provide a source of revenue to the operators and are argued to simplify payouts[3]. In the UK, breakage results in levels of returnable dividends (to the pound) with a constant tick-size of 10 pence[4] for dividends[5] above £1.10. The constant tick-size of 10 pence is antipodal to the bookmakers' price-grid where the tick-size is larger for odds levels greater than 5/4 and smaller for odds levels below evens (1/1)[6]. Finally, in this market, the odds (or dividends) are not known until after the race has started. However, before the start of the race, the tote displays provide an idea of the dividends based on the money bet so far.

This chapter is the first to provide a formal treatment on the effect of the price-grid in a betting market on bettors' returns. It shows that the fixed-odds menu employed by British bookmakers for horse racing betting is a contributory factor to the FL-Bias. In addition, it shows that the price-grid resulting from breakage in pari-mutuel betting can be responsible for the reverse FL-Bias sometimes observed in pari-mutuel betting, consistent with the empirical findings of Coleman (2004)[7].

The intuition underlying the results in this paper can most easily be understood by focusing on the pari-mutuel case. Dividends are usually rounded downwards to the nearest ten pence per pound bet. The relative impact of potentially losing 10p on a bettor's winnings (per pound) on a hot favourite is huge compared to losing 10p on a horse paying large dividends. For example, if the dividends in the absence of breakage should have been £1.19, the bettor will lose 9p (nearly half his profit), compare this to losing 9p on a horse that should have returned £49.99 in the absence of breakage.

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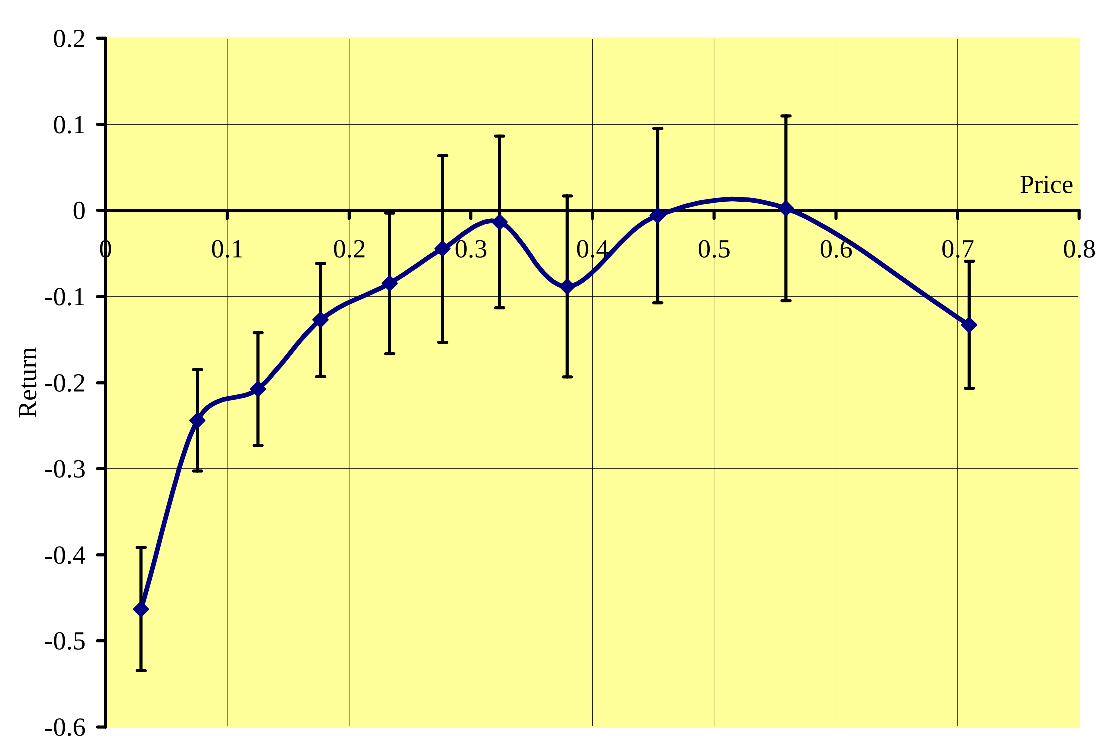
[3] Busche and Walls (2002), page 50.
[4] In the U.S., dividends are quoted to a \$2 stake, so they are rounded down to the nearest \$0.20.
[5] Dividends can be thought of as 'gross' odds, a dividend of £7.00 is equivalent to bookmaker odds of 6/1, so dividends of £1.10 are equivalent to bookmaker odds of 1/10. Below dividends of £1.10, the dividends are £1.05, £1.02 and (presumably) £1.01, however these levels are rarely returned (if indeed at all for win bets). It is possible for a horse to return a dividend of £1.00.
[6] See Figures 4.4 and 4.5 where the widths between the points for prices above 0.5 are much wider for tote prices.
[7] Busche and Walls (2001) are the first to point out that ignoring the effect of breakage may bias statistical tests based on paid-out dividends towards rejection of the hypothesis of efficient betting markets.

After reviewing the literature on broken odds (Section 4.2) and discussing the price-grid that bookmakers in the UK employ (Section 4.3), a formal analysis of the link between the price-grid and expected returns from betting is provided. Fixed-odds markets (Section 4.4) are modeled using risk neutral bookmakers who know the 'true' win-probabilities of the horses in the race. They vie for bets subject to the constraint that posted odds are from the menu of 'quotable' odds. Bettors are assumed to posses no insider information and there is a positive probability that each horse in a race is bet on. In this environment, a condition for the existence of a FL-Bias is derived. The implications of the model are illustrated using data from the 2003 flat season in the UK in Section 4.6. The actual average returns on bets are compared to the theoretical returns derived from the model. It is found that the model generates a FL-Bias in this context, which is of a similar nature to the actual one observed albeit less extreme. An analogue model is then examined for pari-mutuel betting (Section 4.5). It is found that the price-grid resulting from breakage in pari-mutuel betting does not satisfy the condition for a FL-Bias derived in Section 4.4. The resulting expected pari-mutuel returns exhibit a reverse FL-Bias, in line with Coleman's (2004) empirical result.

4.2 – RELATED LITERATURE

The FL-Bias can best be understood by looking at a specific example. Figure 4.1 shows the average returns from bets placed at bookmaker starting odds (translated into *prices* for a one pound winning prize) for the 2003 flat season in the UK. The error bars denote 95% confidence intervals for the estimated returns (see Section 2.3 for an explanation). A FL-Bias is present: bets on longshots (low price and therefore high odds horses) yield worse returns on average than those on favourites (high price, low odds), although returns worsen again for the extreme favourites. Similar patterns are well documented for pari-mutuel and bookmaker betting, (e.g. Law and Peel (2002), where for British bookmaker betting, returns also fall dramatically for prices below £0.20; see Figure 2.1). This chapter complements the rationales for the existence of a FL-bias outlined in Chapter 1 by showing what biases may be embedded in the rules governing the quotable odds in bookmaker markets and the dividends in pari-mutuel markets.

FIGURE 4.1
BETTOR RETURNS FROM BETS IN DIFFERENT PRICE RANGES FOR THE 2003 FLAT SEASON IN THE UK



Notes: Returns are the expected loss per £ bet at starting odds. Favourites on the right, and longshots on the left. Error bars denote 95% confidence intervals of the estimated number of winners.

For pari-mutuel markets, Busche and Walls (2001) construct an *index of breakage*, defined as the sum (by race) of the product of the dividends of the horse, D_i and the probability that they will be realized (the bet fraction is used as a proxy): $\sum_i (x_i / w) D_i$, where x_i is the total amount bet on horse *i*, and *w* is the total amount bet on all the horses in that race. Their motivation for using this measure is that horses with low odds will have higher breakage costs, and multiplying by a subjective probability translates this into the expected breakage that will be realized (and ensures that breakage costs increase with odds)[8]. Using Ali's (1977) data from racecourses located in the United States, they classify races using the index of breakage and calculate *z*-statistics (see Section 1.3) for horses of up to the eight favourite (the horse which collects the eight largest share of the pool). Their results are presented in Table 4.1.

[8] Busche and Walls (2001, page 602), show that with track take at 17%, a horse which has 49% of the pool bet on it adds 0.779 to the index of breakage, and a horse collecting 5% of the pool adds 0.053 to the index.

TABLE 4.1

Z-STATISTICS FOR RACES GROUPED BY BUSCHE AND WALLS' INDEX OF BREAKAGE COST

Group	Index of breakage	Favourite position							
		1	2	3	4	5	6	7	8
1	0.1742	0.32	−0.10	−1.46	−1.12	0.51	0.56	1.64	0.79
2	0.1931	2.12	0.75	−1.65	−1.16	−0.10	−0.72	0.03	0.62
3	0.2002	−1.48	−1.58	0.31	−0.55	1.46	0.72	3.48	0.96
4	0.2058	0.20	−0.96	−0.82	1.46	0.60	0.15	0.14	−0.57
5	0.2107	−0.50	−0.75	−0.71	−0.01	−1.24	0.77	3.27	4.03
6	0.2155	−0.45	−0.97	−1.99	1.52	1.41	−0.75	3.31	1.15
7	0.2203	−0.47	0.99	−1.64	−1.19	1.89	−0.14	1.10	1.45
8	0.2250	−1.75	0.19	−0.12	1.66	−0.14	1.90	−0.53	0.23
9	0.2299	−0.35	−0.11	0.71	−0.49	−0.70	−0.36	0.68	2.47
10	0.2349	−1.77	−0.44	1.31	0.47	−0.09	0.81	0.30	1.77
11	0.2404	−3.18	0.91	−0.73	1.09	2.20	0.55	0.65	2.70
12	0.2463	−2.33	−0.18	−0.10	1.34	0.28	1.07	2.16	1.48
13	0.2528	−3.12	−1.21	0.34	2.15	0.80	3.50	1.99	1.07
14	0.2599	−1.64	0.99	0.86	−0.28	−1.25	0.92	1.75	0.52
15	0.2683	−1.50	0.82	−2.52	1.89	1.18	1.79	0.46	1.38
16	0.2783	−1.11	−0.37	−1.04	1.16	1.60	−0.55	1.93	2.18
17	0.2914	−4.93	1.73	0.95	1.13	1.17	1.17	2.38	4.08
18	0.3096	−1.68	−0.29	0.16	2.34	0.29	0.23	−0.03	1.86
19	0.3393	−4.23	0.95	0.97	1.83	2.09	0.34	1.99	2.33
20	0.5086	−7.18	2.15	2.91	2.12	4.35	3.29	0.25	2.66

Notes: z-statistics are for the null hypothesis of equalized returns across favourite positions. Each group consists of 1012 races. Source: Busche and Walls (2001).

Table 4.1 shows that in races where the index of breakage is higher, the FL-Bias is stronger (note positive *z*-statistics are consistent with over-betting and negative *z*-statistics are consistent with under-betting). However, this is against the earlier intuition that breakage should cause returns to exhibit a reverse bias because the impact of breakage is more harmful to the dividends of favourites. The reason for the contradiction is that *favourite position* is not comparable across different groups of index of breakage. In groups with a high index of breakage, the runners are likely to be mainly favorites. For example, the eight favourite is likely to have a relatively high implied win-probability compared to an eight favourite in a low breakage group. Hence, from Table 4.1, it is not possible to compare the extent of the FL-Bias in the way it is defined here (based on position of favouritism and not on price) across races with different indices of breakage. A measure such as that proposed in Chapter 2, or *z*-statistics sorted by implied win-probability (as used in Chapter 2) would be more appropriate.

Coleman (2004) was the first to explicitly point out the link between breakage and the reverse FL-Bias, arguing that rounding downwards “has the greatest proportional impact on short priced horses, it brings relatively greater reduction in return from backing favourites and so weakens the longshot bias” (page 320). He estimates the returns from Busche and Hall’s (1988) data (which exhibits a reverse FL-Bias) by rounding the (pre-breakage) dividends down to the nearest 10 and 20 cents, and finds that the greater the amount of rounding, the stronger the reverse FL-Bias is.

The impact of breakage on Japanese and Hong Kong pari-mutuel odds data is investigated by Walls and Busche (2003). When using the broken odds data which ranks each horse by their position of favouritism in each race, the data exhibits a minor FL-Bias. The Japanese data exhibits a small FL-Bias, whereas the Hong Kong data exhibits a ‘first-favourite underbet, fourth-favourite overbet’ bias. They find that when using the exact data based on the actual shares of the pool of each horse, the FL-Bias almost completely disappears. In accord with Busche and Walls (2001), this empirical result again suggests that breakage causes a FL-Bias and actually contradicts the empirical result of Coleman (2004). Together with the results from this chapter this would suggest that a reverse FL-Bias embedded in the price-grid is overturned by other factors that influence prices/odds.

In this chapter, the conditions for the price-grid to impose a FL-Bias onto the quoted odds in bookmaker markets are derived. In contrast to British bookmaker odds, the price-grid arising from pari-mutuel dividends does not satisfy these conditions, pushing returns towards exhibiting a reverse FL-Bias. An implication of this finding is that when using broken odds data, bookmaker and pari-mutuel markets have different 'built-in' biases, so comparisons across these markets have to be interpreted with caution. An example of such a cross-market analysis is Gabriel and Marsden's (1990) examination of betting market efficiency. They compare the returns from the same winning bets placed on the tote (pari-mutuel) and with bookmakers and found them to be significantly different. They used the following specification:

$$T_i = c + bSP_i + u_i \,, \tag{4.1}$$

where T_i is the tote payout for the winning bet in race i, SP_i is the bookmaker payout for the winning bet at starting odds in race i[9]. To test for market efficiency (or equality of the returns from the two systems), they test the joint hypothesis that $c = 0$ and $b = 1$, this is always rejected at the 10% level. The signs and magnitudes of the estimated parameters vary depending on which sub-sample is taken. For all races, they find c to be negative and b to be greater than 1, this indicates that bookmaker markets pay better for favourites and less generously for longshots[10]. A reason for this result is that pari-mutuel payouts have no upper limit; a sole winning ticket can scoop the whole (taxed) pool. In sub-samples they impose a tote returns ceiling and the estimates of c are typically positive (around 20) and the estimates of b are typically around 0.7 to 0.8. These estimates curiously suggest that the returns on favourites are far superior on the pari-mutuel, and that this gap closes slowly as odds increase[11]. Bruce and Johnson (2000) conduct an investigation exploring the FL-

[9] When running a regression of equation (4.1) the regression line will pass through the mean tote and bookmaker odds. This automatically assigns the point where the returns from the two systems are equal and where returns are superior or inferior for the two systems.

[10] It turns out that part of Gabriel and Marsden's data was incorrect, see Gabriel and Marsden (1991). Some of the observations for tote returns are twice as large as they should have been. This means that their estimates of the parameters in (4.1) were incorrect. The regression estimates of the true data are not supplied in Gabriel and Marsden (1991), they merely state that the results of the conclusions drawn from the joint-hypothesis tests remain the same.

[11] This observation could be due to the errors in some of the observations mentioned in footnote 72.

Bias present in bookmaker and the parallel tote market for UK data from 1996. They find a FL-Bias in the bookmaker market but not the tote market where the probabilities implied by the dividends are accurate estimates of the true win-probability. They also find that bookmakers offer better payouts compared with the tote for favourites (horses with starting odds of less than 5/2). For horses with starting odds greater than 5/1, tote returns are found to be superior, supporting Gabriel and Marsden's main finding.

4.3 – BRITISH BOOKMAKER ODDS

In theory, bookmakers are free to set any odds levels they wish to: there is no 'official' list of odds that bookmakers at the racecourse are obliged to set quotes from[12]. However, in practice, a specific list of quoted odds prevails (shown in Table 4.2 and in more detail in the Appendix Table 4.A.1). The price-grid observed is influenced by the old (pre-decimalisation) British currency. Hence, there are many quotes with 4 and 8 as the denominator. These are the odds that will be returned as starting odds, and the odds that one will hear quoted on the news, e.g. "the winner of the Derby was returned at 9/2". One will not observe bookmaker odds returned at for example 4.9/1 or an equivalent level. Table 4.2 shows that the tick-size expressed in terms of prices actually becomes smaller as odds increase, it is around 0.02 pence for low odds levels, and less than 0.01 for odds levels above 7/1. Most of these odds levels have been handed down for many generations since the early nineteenth century when horse race betting markets took their current shape.

There is nothing (legally) stopping bookmakers from offering any odds levels they like. This is important to note. If the betting market in each period is a truly competitive spot market, one would expect bookmakers to try to undercut each other slightly to capture the whole betting market. The practice however, is inconsistent with this. There are absolutely no deviations from the pricing grid[13] in over thirty nine thousand observations of quoted odds for the UK 2003 flat season - and one

[12] It is the odds from this market that the odds in the betting shops are reflecting during the market phase. Also the starting odds are determined by this market.

[13] There is one instance of starting odds of 15/1 being returned at Doncaster racecourse on the 22nd of March 2003. However, it is more likely that the starting odds adjudicators failed to agree on whether the starting odds should have been 14/1 or 16/1

would not expect that all horses have expected winning probabilities corresponding to exactly these odds. This begs the question ‘how is the price grid sustained?’

Before addressing this, note first that a price-grid implies that bookmakers make profits from at least some of the bets they lay. These may be required to offset fixed costs and provide bookmakers with incentives to ‘make the market’ at the racecourse – thus the grid would serve a similar purpose as minimum tick-size regulations in financial markets.

TABLE 4.2

LIST OF QUOTED ODDS ABOVE EVENS USED BY BRITISH ON-COURSE BOOKMAKERS

11/10	(-0.024)	**3/1**	(-0.017)	**9/1**	(-0.005)	**50/1**	(-0.005)
6/5	(-0.022)	**10/3**	(-0.019)	**10/1**	(-0.009)	**66/1**	(-0.005)
5/4	(-0.010)	**7/2**	(-0.009)	**11/1**	(-0.008)	**80/1^**	(-0.003)
11/8	(-0.023)	**4/1**	(-0.022)	**12/1**	(-0.006)	**100/1**	(-0.002)
6/4	(-0.021)	**9/2**	(-0.018)	**14/1**	(-0.010)	**125/1**	(-0.002)
13/8	(-0.019)	**5/1**	(-0.015)	**16/1**	(-0.008)	**150/1**	(-0.001)
7/4	(-0.017)	**11/2**	(-0.013)	**18/1^**	(-0.006)	**200/1**	(-0.002)
15/8	(-0.016)	**6/1**	(-0.011)	**20/1**	(-0.005)	**250/1**	(-0.001)
2/1	(-0.014)	**13/2**	(-0.010)	**22/1^**	(-0.004)	**300/1**	(-0.001)
85/40	(-0.013)	**7/1**	(-0.008)	**25/1**	(-0.005)	**400/1**	(-0.001)
9/4	(-0.012)	**15/2**	(-0.007)	**28/1^**	(-0.004)	**500/1**	(-0.000)
5/2	(-0.022)	**8/1**	(-0.007)	**33/1**	(-0.005)		
11/4	(-0.019)	**17/2^**	(-0.006)	**40/1**	(-0.005)		

Inverses are also quoted for ‘odds-on’ competitors. Changes in the price from the interval above (the *tick-sizes* in terms of prices) are in parentheses. ^Denotes odds levels which appeared recently and are quoted often. Source: personal observation, consultation with on-course bookmakers Chris Ralph and Tony Lusardi, and Professor Leighton Vaughan Williams.

Christie and Schultz (1994) show that even with minimum tick-size regulation in place, dealers in the NASDAQ market sustained a *different* price-grid. The market makers could set quotes with a tick-size of an eighth of a dollar for quotes above $10, but in practice avoided odd-eighth quotes, i.e. most of the time, the grid consisted of even-eighth quotes. After the publication of these revelations, odd-eighth quotes became more frequent and the tick-size was reduced by regulators to one cent in 2000. Christie, Harris and Schultz (1994) propose odd-eighth quotes avoidance was sustained through tacit collusion. The argument essentially is that dealers are involved in a repeated game and deviations from the agreed price-grid are easily observable, so that a trigger strategy mechanism can sustain the agreement[14]. Bookmaker markets and the pre-reform NASDAQ market share many of the structural features of which Christie, Harris, and Schultz (1994) list as being conducive to sustaining a price grid: bookmakers are playing a repeated game and their quotes are transparent. There is also a degree of camaraderie amongst bookmakers.

It is also interesting that in the last few years, some new odds levels have been quoted, e.g. 17/2 and 18/1. This supports somewhat the tacit agreement hypothesis: in the absence of exchanges, deviations from the grid on horse *n*'s odds can cause exposure to risk because offering slightly better odds can attract many bets on horse *n*. With the advent of betting exchanges, bookmakers can hedge by backing horse *n* (usually at superior odds) on the exchanges. If bookmakers are averse to big risks, the availability of exchanges is likely to increase the potential profits from deviating from the price grid. A finer grid helps reduce these deviation gains and sustain the agreement.

The introduction of *computerised tickets* in 1998, has led to coordination on a list of quotable odds. Bookmakers operate a terminal[15], which displays their odds quotes and prints tickets. Each computer system has its own pre-programmed list of odds which cannot be overridden easily. Bookmakers adjust the odds they are offering by hitting a '+' or a '-' after selecting the horse whose odds they wish to alter. There are at least four popular computer systems (for example the Winning Odds, Exante and Futurebet systems) used on racecourses. While the pre-programmed odds vary slightly across the different systems, each system has the 'main' grid pre-programmed into it. Some systems may offer additional quotes, particularly the newer systems. For example, one bookmaker operating System X, lengthening the odds of a horse which is currently at 8/1 will be obliged to adjust the horse's odds to

[14] Cason (2000) proposes that dealers were *explicitly* colluding and backed up his claim with experimental evidence. He set up a dealer experiment in which the subjects are not allowed to communicate in one treatment, and another treatment where they can. Cason found that collusion is present in the treatment where communication was allowed; the treatment which reflected the real market.

[15] These terminals differ in terms of how advanced (and hence how expensive) they are, their basic functions also include displaying the bets already taken, and potential profits and losses.

9/1, but another bookmaker, operating System Y, wishing to increase the 8/1 horse's odds by an increment can adjust the odds to 17/2.

In a market context, the reality is a little more complicated than this because if most of the other bookmakers are adjusting their odds to 9/1, it is likely that the bookmaker (with the option to adjust his odds to 17/2) will also follow suit because odds of 17/2 are not as competitive, particularly if the bookmaker needs to lay bets on that horse to 'balance his books'.

4.4 – THE IMPACT OF THE PRICE-GRID ON RETURNS: BOOKMAKER MARKETS

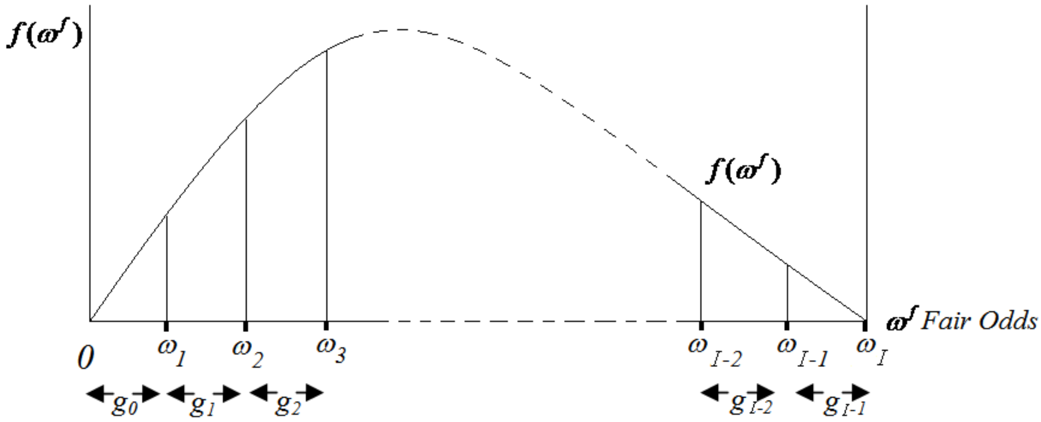
In this section, the theoretical returns from bets are characterised. Conditions for a FL-Bias will then be derived in terms of odds, ω, and then prices, p[16]. There are $n>2$ risk neutral bookmakers who know the 'true' win-probabilities of the horses in the race. Assume that the fair odds reflecting these winning probabilities have a density function $f(\omega^f)$, where $\omega^f \in \Omega^f$, and $\Omega^f = [0, \infty]$ is the set of fair odds. Bookmakers are restricted with respect to the odds they can offer bettors, the odds quoted must be on the price-grid, denoted Ω. These odds levels will be referred to as the *quotable*/*quoted* odds: $\Omega = \{\omega_1, \omega_2, ..., \omega_I\}$, where ($\omega_1 < \omega_2, ... < \omega_I$). Bettors are assumed to possess no inside information and there is positive probability that each horse in a race is bet on.

Bookmakers are assumed to be risk neutral, profit maximising and rational. They will offer 'fair' bets, if allowed. There are no insider traders in this market so bookmakers do not need to set odds with a FL-Bias incorporated in them as they do in the Shin (1991 and 1992) and Ottaviani and Sørensen (2005) model. However, the price-grid prevents them from always setting fair odds. Because a bookmaker will not offer odds which result in an expected loss for him, he always rounds downwards; so a horse with $\omega_i \leq \omega^f < \omega_{i+1}$ will have quoted odds of ω_i. Figure 4.2 illustrates this using a scenario where the density of the distribution of fair odds

[16] A bet struck at odds of $\omega/1$ will have a price, $p = 1/(\omega + 1)$

increases and then decreases. For an idea of the actual distribution of fair odds, see Appendix 4.A.1 for a discussion of the actual quoted odds.

FIGURE 4.2
THE PRICE-GRID AND THE DISTRIBUTION OF FAIR ODDS



ω^f = fair odds
ω_i = quotable/quoted odds
g_i = gap between odds level *i* and *i+1* (tick-size)

Two definitions of the FL-Bias are used in the formal analysis. A *strict FL-Bias* exists if bettor returns per unit stake increase monotonically with price (that is decrease monotonically with odds). In other words, when moving to the next level of quotable odds on the grid (i.e. from ω_i to ω_{i+1}), returns fall monotonically. A *loose FL-Bias* exists if there is a threshold level of odds, so that the average return above (the longshots) is lower than that for odds below the threshold (the favourites). That is, the returns from backing favourites are superior to the returns from backing longshots, but returns need not increase monotonically with prices. The relationship depicted in Figure 4.1 would be that of a loose FL-Bias because returns improve on average with prices; e.g. the returns of bets struck at just below a price of £0.40 yield inferior returns to bets struck at prices of around £0.30. However, returns do not improve monotonically with prices and also the returns on extreme favourites are inferior to the returns on favourites.

Given the environment under consideration, the expected profit for a bettor, π, from a £1 bet on a horse quoted at ω_i will be the sum of the horses' odds multiplied by its expected probability of winning given its quoted odds, and the expected probability of making a loss of £1:

$$E[\pi \mid \omega_i] = E[p^f \mid \omega_i] \times \omega_i + \left[1 - E[p^f \mid \omega_i]\right] \times (-1) \,, \tag{4.2}$$

where

$$E[p^f \mid \omega_i] = \frac{1}{E[\omega^f \mid \omega_i] + 1} \,. \tag{4.3}$$

There are two effects at work. First, the larger the tick-size (g), the greater (expected) fair odds are compared to the odds quoted, hence the greater the potential losses will be for the bettor. This shall be referred to as the *tick-size effect*. Secondly, there is the *distribution of odds effect*, which relates to the impact of the way the odds are distributed. With an upward sloping fair odds distribution, the expected fair odds for a given ω_i will be greater than the corresponding value for the hypothetical case of uniformly distributed odds[17]. Thus, bettor returns are more negative than they would otherwise be compared to uniformly distributed fair odds if the distribution of odds is upward sloping over a particular interval. However, for downward sloping parts, the difference between the quoted odds and the fair odds will be smaller and bettor returns will be superior compared to the returns with uniformly distributed fair odds. This effect will be investigated in more detail in Appendix 4.A.1 where the distribution of (quoted) odds observed in the 2003 flat season is compared to the situation with uniformly distributed odds. It turns out that for sensible odds levels, uniformly distributed odds provide a reasonable approximation.

The condition for a strict FL-Bias is computed for the general case, and then illustrated for the cases of uniformly distributed odds (this section) and a constant tick-size (Section 4.5). It is convenient to describe distributions of fair odds in terms of the properties of the conditional expectation that gives the expected fair odds of a horse with quoted odds of ω_i. Let the tick-size, $g_i = \omega_{i+1} - \omega_i$. A parameter a_i can be defined as follows:

[17] Consider the fair odds interval from 10/1 to 11/1, if fair odds are uniformly distributed, $E[\omega^f|\omega_i=10]=10.5$. Any upward sloping distribution of fair odds (or a distribution such that the distribution of fair odds was concentrated near 11/1) would yield an expected value of greater than 10.5/1. Hence there is a smaller chance of success but the payoff under the state of success is the same.

$$E[\omega^f \mid \omega_i] = E[\omega^f \mid \omega_i \le \omega^f < \omega_{i+1}] = \omega_i + a_i g_i \,. \tag{4.4}$$

That is, the expected value of the fair odds of horse *n* given that it is quoted at ω_i is a fraction a_i of the way along interval *i* (for uniformly distributed fair odds, $a_i = \bar{a} = ½ \;\; \forall\, i \in 0,...,I-1$). So $a_i g_i$, denotes the absolute distance across a tick of where the expected value of the fair odds lies, in other words the *effective* tick-size. This is an absolute measure of how much *value* is taken away due to the discrete nature of the odds. It is important to note that (4.4) reflects the way most studies analyse the FL-Bias: a large sample of horses is collected and then for each horse the expected fair odds (based on the sample average win-probability in that odds range) is compared with its quoted odds. A bettor might have a different estimate than that given here because he can use the information contained in other horses' odds as well to form expectations[18]. Putting (4.3) and (4.4) together yields:

$$E[p^f \mid \omega_i] = \frac{1}{\omega_i + a_i g_i + 1} \tag{4.5}$$

Substituting into (4.2), the expected returns from placing bets at odds of ω_i becomes:

$$E[\pi \mid \omega_i] = \frac{\omega_i}{\omega_i + a_i g_i + 1} - \left[1 - \frac{1}{\omega_i + a_i g_i + 1}\right],$$

$$E[\pi \mid \omega_i] = \frac{\omega_i}{\omega_i + a_i g_i + 1} - \left[\frac{\omega_i + a_i g_i + 1}{\omega_i + a_i g_i + 1} - \frac{1}{\omega_i + a_i g_i + 1}\right],$$

$$E[\pi \mid \omega_i] = \frac{-a_i g_i}{\omega_i + a_i g_i + 1}\,. \tag{4.6}$$

Expression (4.6) demonstrates that bettors expected returns will be negative, as expected.

[18] The relation between the odds offered on a particular horse and those offered on the other horses in the race is explored by Collier and Peirson (2005)

PROPOSITION 1: *A strict FL-Bias exists if and only if:*

$$\forall\ i \in \{0,\ldots,I-1\} \qquad \frac{a_{i+1}g_{i+1}}{a_i g_i} > \frac{\omega_i + g_i + 1}{\omega_i + 1}. \tag{4.7}$$

Proposition 1 states that a strict FL-Bias exists if the relative gain in the gross quotable odds when moving up a tick is less than the relative gain in the absolute distance across the tick of the expected value of the odds when moving up a tick. In other words, the effective tick-size ($a_i g_i$) needs to grow at a faster rate than the gross quotable odds (ω_i+1).

Proof: A strict FL-Bias exists if for all odds intervals the expected return on a competitor quoted at odds of ω_i is greater than that for a competitor quoted at odds ω_{i+1} :

$$E[\pi \mid \omega_i] > E[\pi \mid \omega_{i+1}]$$

$$\Leftrightarrow \frac{-a_i g_i}{\omega_i + a_i g_i + 1} > \frac{-a_{i+1}g_{i+1}}{\omega_{i+1} + a_{i+1}g_{i+1} + 1}. \tag{4.8}$$

To find the condition, use the fact that:

$$g_i = \omega_{i+1} - \omega_i \qquad \text{or} \qquad \omega_{i+1} = \omega_i + g_i . \tag{4.9}$$

Solving yields condition (4.7).

Q.E.D.

To gain some intuition for the distribution of odds effect, *a*, the condition for a FL-Bias is examined for the case of any distribution where *a* is constant (for example, uniformly distributed fair odds). For uniformly distributed fair odds where $f(\omega^f)=\overline{C}$, and $a_i = \overline{a}\ \forall\ i \in 0,\ldots,I-1$, condition (4.6) simplifies to (4.10):

$$E[\pi \mid \omega_i] = -\frac{g_i}{2\omega_i + g_i + 2}. \tag{4.10}$$

Condition (4.10) shows that bettors' expected profits decrease with the tick-size[19]; this is the tick-size effect. Rewriting condition (4.7) with constant *a* yields the following corollary to Proposition 1.

COROLLARY 1: *Suppose* $a_i = \bar{a}\ \forall\ i \in 0,\dots,I-1$, *returns then exhibit a strict FL-Bias if and only if:*

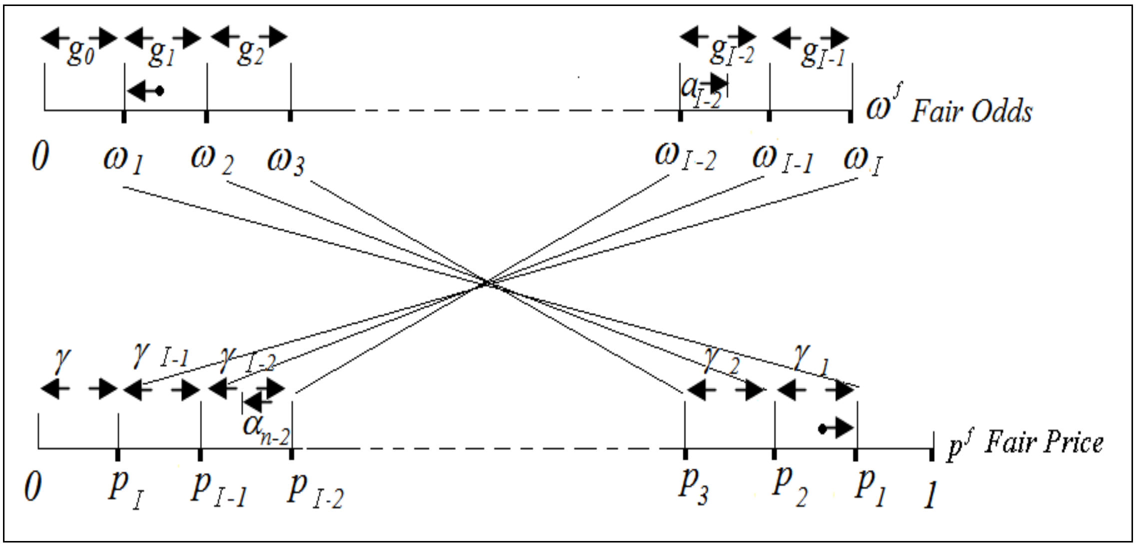
$$\frac{g_{i+1}}{g_i} > \frac{\omega_{i+1}+1}{\omega_i+1} \qquad (4.11)$$

That is, if the distribution of fair odds has a constant *a*, *returns exhibit a strict FL-Bias if the relative gain in the tick-size is greater than the relative gain in the gross quotable odds.* The impact of the distribution of odds effect is explored further in the Appendix (Section 4.A.1) where the distribution of quoted odds (ω_i) is used as a proxy for the distribution of fair odds (ω^f). The distribution of ω_i is then approximated to be a simple function from which the *a*'s can be calculated using conditional expectations. Using this method, it is found that given the distribution of the quoted odds observed in the 2003 flat season, the *a's* do not deviate too much from the value of 0.5. For the distribution across the whole grid, $0.39 < a < 0.57$, and when only sensible odds of between 1/10 and 100/1 are considered, the range falls to $0.45 < a < 0.51$, i.e. the differences arising from the distribution of actual odds from uniformly distributed odds are negligible.

According to Figure 4.1, the actual data from the 2003 flat season in the UK exhibits a FL-Bias in bookmaker betting. Moreover, when moving from the extreme favourites to horses with lower prices, the *returns decline at an increasing rate*. The remainder of this section investigates what nature of the price-grid would lead to this phenomenon under the environment considered. So far, the expected returns have been estimated in terms of odds, and Figure 4.1 is drawn in terms of prices, so the results need to be restated in terms of prices. The situation is depicted in Figure 4.3, the *i*'s for the price interval/tick correspond to the respective odds interval/tick.

[19] Because $\frac{\partial}{\partial g_i} E[\pi \mid \omega_i] < 0$

FIGURE 4.3
DIAGRAMMATICAL REPRESENTATION OF THE PRICE-GRID FOR ODDS AND PRICES



Let p^q denote the set of quotable prices from the set of fair prices or probabilities (p^f):

$$p^f \in [0,1],$$

$$p^q \in [p_1, p_2, ..., p_{I-1}, p_I], \qquad (p_1 > p_2, \ldots > p_I) \tag{4.12}$$

A horse with fair price (win-probability) within the interval associated with γ_i will have a quoted price p_i (it will be quoted at the higher price). The α_is correspond to the proportional distance across an interval the expected value of the fair-price lies (to the left of p_i).

$$E[p^f \mid p_i] = E[p^f \mid p_{i+1} > p^f \geq p_i] = p_i - \alpha_i \gamma_i \tag{4.13}$$

The relationship between a and α, and g and γ are investigated below. Now, γ_i is given by:

$$\gamma_i = p_i - p_{i+1}. \tag{4.14}$$

Using (4.14), (4.9) and that fact that $p = 1/(\omega+1)$, the relationship between γ and g is:

$$\gamma_i = \frac{g_i}{(\omega_i + 1)(\omega_{i+1} + 1)}. \tag{4.15}$$

The relationship between expected odds and prices given their quoted value is:

$$E[p\,|\,p_i] = \frac{1}{E[\omega\,|\,\omega_i\,]+1}\,. \tag{4.16}$$

The relationship between a and α can be established if (4.13) and (4.16) are combined with (4.4). Equating (4.13) with (4.16), and substituting (4.4) into it:

$$p_i - \alpha_i\gamma_i = \frac{1}{\omega_i + a_i g_i + 1}\,. \tag{4.17}$$

Again, using the fact that $p = 1/(\omega+1)$ to substitute for p_i:

$$\frac{1}{\omega_i + 1} - \alpha_i\gamma_i = \frac{1}{\omega_i + a_i g_i + 1} \Rightarrow \frac{1}{\omega_i + 1} - \frac{1}{\omega_i + a_i g_i + 1} = \alpha_i\gamma_i\,. \tag{4.18}$$

Substituting for γ_i from (4.15) yields:

$$\frac{1}{\omega_i + 1} - \frac{1}{\omega_i + a_i g_i + 1} = \alpha_i \frac{g_i}{(\omega_i + 1)(\omega_{i+1} + 1)} \Rightarrow$$

$$\frac{(\omega_i + a_i g_i + 1) - (\omega_i + 1)}{(\omega_i + 1)(\omega_i + a_i g_i + 1)} = \alpha_i \frac{g_i}{(\omega_i + 1)(\omega_{i+1} + 1)} \Rightarrow \frac{a_i g_i}{(\omega_i + a_i g_i + 1)} = \alpha_i \frac{g_i}{(\omega_{i+1} + 1)}\,. \tag{4.19}$$

So:

$$\alpha_i = \frac{a_i(\omega_{i+1} + 1)}{[\omega_i + a_i g_i + 1]}\,. \tag{4.20}$$

Substitute in $g_i = \omega_{i+1} - \omega_i$:

$$\alpha_i = \frac{a_i(\omega_{i+1} + 1)}{[\omega_i + a_i(\omega_{i+1} - \omega_i) + 1]}\,. \tag{4.21}$$

Given (4.21) and the price-grid of odds employed by bookmakers, if a is stable at 0.5, α is also stable at between 0.5 and 0.58[20]. In other words, the difference arising from the distribution of fair prices effect when comparing to uniformly distributed odds is negligible. The expected profit for a bet placed at ω_i will be $-\alpha_i\gamma_i$, so the profit per pound bet on a horse with price p_i will be:

[20] Additionally, using estimates of a approximated from the actual *distribution of quoted odds*, α is stable at between 0.5 and 0.53 for sensible levels of odds, see Appendix 4.A.1.

$$E[\pi \mid p_i] = -\frac{\alpha_i \gamma_i}{p_i}. \tag{4.22}$$

The analogue to condition (4.7) in Proposition 1 expressed in terms of prices is:

$$\frac{\alpha_i \gamma_i}{p_i} < \frac{\alpha_{i+1}\gamma_{i+1}}{p_{i+1}} \qquad \text{or} \qquad \frac{\alpha_i \gamma_i}{\alpha_{i+1}\gamma_{i+1}} < \frac{p_i}{p_{i+1}}, \tag{4.23}$$

where $p_i > p_{i+1}$. A FL-Bias will be present if the absolute distance across a tick where the expected value for the fair price ($\alpha_i\gamma_i$) is, i.e. the effective tick-size in prices, grows slower than the quotable prices. With α constant, quotable prices need to grow at a faster rate than the tick-size of prices as prices increase.

COROLLARY 2: *Suppose* $\alpha_i = \overline{\alpha}\ \forall\ i \in 0,\dots,I-1$, *then when moving from the extreme favourites to horses with lower prices, the expected returns decline at an increasing rate if:*

$$g_{i-1} < \frac{\gamma_{i+1}}{p_{i+1}\gamma_i} - \frac{\gamma_i}{p_i\,\gamma_{i-1}}. \tag{4.24}$$

Proof: The condition for returns to decrease at an increasing rate as price falls can be written as:

$$\frac{E[\pi \mid p_i] - E[\pi \mid p_{i+1}]}{\gamma_i} > \frac{E[\pi \mid p_{i-1}] - E[\pi \mid p_i]}{\gamma_{i-1}}. \tag{4.25}$$

Substituting in (4.22) for the expected returns yields:

$$\frac{\alpha_i\gamma_i}{p_i\,\gamma_i} - \frac{\alpha_{i+1}\gamma_{i+1}}{p_{i+1}\gamma_i} < \frac{\alpha_{i-1}\gamma_{i-1}}{p_{i-1}\gamma_{i-1}} - \frac{\alpha_i\gamma_i}{p_i\,\gamma_{i-1}} \qquad \text{or} \qquad \frac{\alpha_i}{p_i} - \frac{\alpha_{i-1}}{p_{i-1}} < \frac{\alpha_{i+1}\gamma_{i+1}}{p_{i+1}\gamma_i} - \frac{\alpha_i\gamma_i}{p_i\,\gamma_{i-1}} \tag{4.26}$$

With a constant value of α:

$$\frac{1}{p_i} - \frac{1}{p_{i-1}} < \frac{\gamma_{i+1}}{p_{i+1}\gamma_i} - \frac{\gamma_i}{p_i\,\gamma_{i-1}}. \tag{4.27}$$

The absolute change in the reciprocal of quotable price (the tick-size in terms of odds) has got to be less than the absolute change in the relative decline of the tick-size in terms of price (normalised by price). This is the same as (4.24).

Q.E.D.

4.4 – THE IMPACT OF THE PRICE-GRID ON RETURNS: PARI-MUTUEL MARKETS

In contrast to fixed-odds betting where numerous bookmakers compete for bets, there is usually a monopoly in pari-mutuel (tote) betting. As mentioned earlier, in pari-mutuel markets, after the total amount bet is taxed (the track-take) and divided by the number of winning tickets, the dividends are rounded down to the nearest 10 pence (in the UK), so called breakage, to form the final dividends. Breakage causes pari-mutuel dividends to have a constant tick-size of 10 pence. The results in the previous section can thus be extended to investigate the effects caused by the nature of the pari-mutuel price-grid. The following analysis looks at the effect of a constant tick-size in the bookmaker markets setting. The results apply directly to a pari-mutuel setting if bettors place bets in proportion to the true win-probabilities of the runners – an assumption that underlies many analyses of market efficiency in pari-mutuel betting (e.g. see the works of Busche, and Bruce and Johnson (2000)), and in the absence of track-take. This ensures that the (pre-breakage) dividends will reflect the fair odds (plus one).

COROLLARY 3: *Suppose that the tick-size is constant so that* $g_i = \bar{g} \ \forall \ i \in 0,\ldots,I-1$, then *returns will usually exhibit a reverse FL-Bias.*

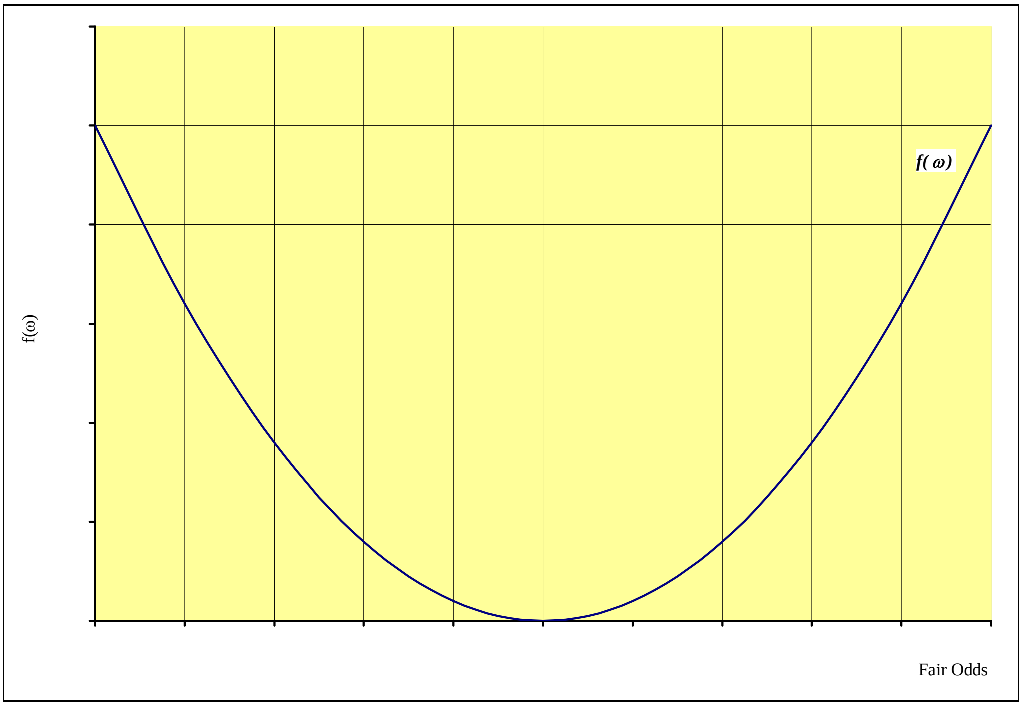
Proof: Condition (4.7), with a constant value of g becomes:

$$\frac{a_{i+1}}{a_i} > \frac{\omega_{i+1}+1}{\omega_i+1}. \tag{4.28}$$

This implies that in order for returns to exhibit a FL-Bias with a constant tick-size, a needs to grow faster than the returnable dividends. This condition is difficult to satisfy because the right hand side will always be greater than unity, this means that *a* has to increase for every successive odds interval. For the parameter *a* to be growing in Ω requires a (very) convex distribution of fair odds. $f(\Omega)$ needs to either: 1) fall at a decreasing rate as odds increase, 2) increase at an increasing rate as odds increase, or 3) a combination of the two with the fall before the rise. Such a relationship is shown in Figure 4.4. This is a *necessary*, but *insufficient* condition and is highly unlikely. Under the first situation, the majority of horses will be hot favourites, which is not possible (since probabilities need to sum up to one). Under the second situation, the majority of horses will be extreme longshots with favourites a rarity. And under the third situation, there will be more horses which are extreme favourites or longshots, with few horses in the middle of the odds spectrum, this is the opposite of what is observed in reality, (see Appendix 4.A.1: Figure 4.A.1). This indicates that a *pari-mutuel price-grid cannot cause returns to exhibit a FL-Bias*, it causes returns to exhibit a reverse FL-Bias. This goes against the findings of Busche and Walls (2001 and 2003) which suggest that breakage contributes towards a FL-Bias[21].

FIGURE 4.4

A DEPICTION OF THE DISTRIBUTION OF FAIR ODDS REQUIRED FOR A PRICE-GRID WITH A CONSTANT TICK-SIZE TO CAUSE RETURNS TO EXHIBIT A FAVOURITE-LONGSHOT BIAS



[21] Busche and Walls do not provide a reason towards the cause of what they observe.

Now consider the case where both *a* and the tick-size *g* are constant.

COROLLARY 4: *Suppose that* $a_i = \bar{a} \; \forall \; i \in 0,\ldots,I-1$ *and* $g_i = \bar{g} \; \forall \; i \in 0,\ldots,I-1$, *then expected returns always exhibit a strict reverse FL-Bias.*

Proof: With a distribution of odds where *a* is constant and a constant tick-size; as there is in the pari-mutuel grid, (4.7) becomes:

$$\bar{g} > \frac{\bar{g}(\omega_i + \bar{g} + 1)}{(\omega_i + 1)} \Leftrightarrow (\omega_i + 1) > \omega_i + \bar{g} + 1$$
$$\bar{g} < 0 \,. \tag{4.29}$$

Condition (4.29) cannot be satisfied, so with a constant tick-size, returns will increase monotonically with odds[22]. The price-grid caused by breakage steers returns towards exhibiting a reverse FL-Bias.

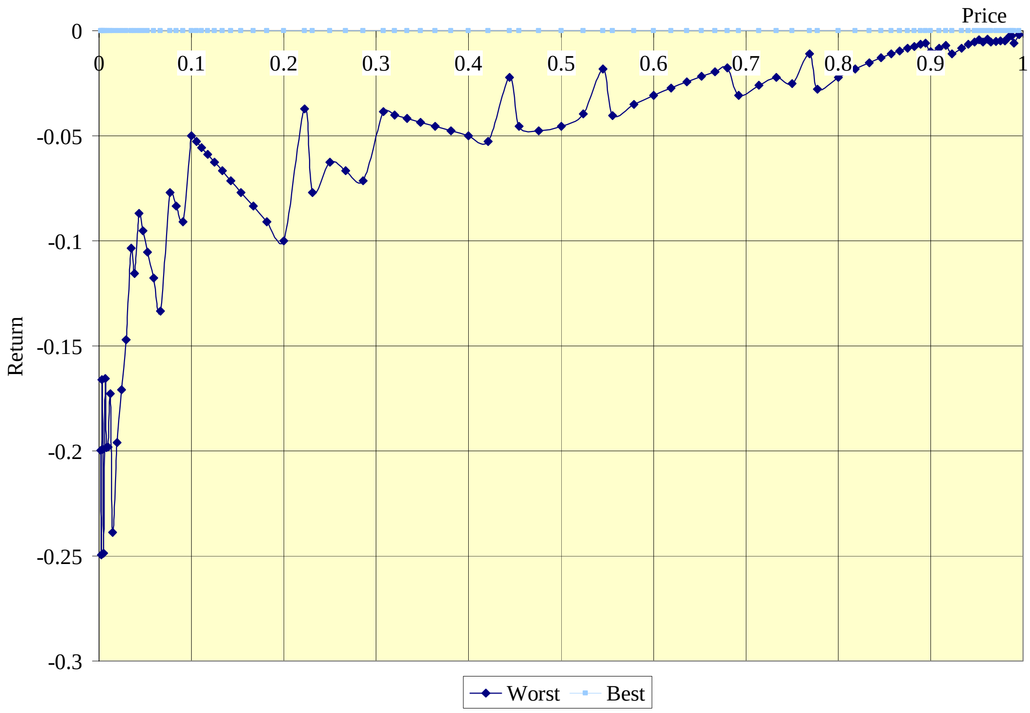
The conditions for a strict FL-Bias under different circumstances have been derived for bookmaker markets in Section 4.3, and for pari-mutuel betting in Section 4.4. The remainder of this chapter focuses on British bookmaker betting by verifying whether the model predicts a (loose) FL-Bias when using the price-grid outlined in Table 4.2. Additionally, the same issue will be explored for pari-mutuel betting to see if the model predicts a (loose) reverse FL-bias when using the price-grid implied by breakage, followed by a comparison of the returns generated by the two grids.

4.6 – THEORETICAL AND ACTUAL RETURNS GIVEN THE PRICE-GRIDS

In this section, the theoretical returns of bets given by the bookmaker and pari-mutuel price-grids are compared. Consider the environment in Section 4.4 with rational risk neutral, profit maximising bookmakers, who know the true winning probabilities of all the horses, but are only able to set the odds in Table 4.2. The possible range of returns for bettors is presented in Figure 4.5.

[22] At least for dividends above £1.10.

FIGURE 4.5
THEORETICAL LIMITS OF BETTOR RETURNS BASED ON THE BOOKMAKER PRICE-GRID



The limits of the returns in Figure 4.5 are constructed based on the best and worst possible bettor returns given a horse's price in the environment being considered. For example, if ω^f is 9/1 (fair-price = 0.1) and the quotable odds below 9/1 are 17/2 (though it is more likely to be 8/1 because 17/2 is one of the new quotes), in the best case scenario, ω_i = 9/1, hence expected returns will be zero, in the worst case scenario, ω_i = 17/2 and the expected loss will be 5p per £1 bet, (if ω_i was 8/1, the expected losses are 5.6p). The unobserved actual expected returns will lie in between the two lines, and given that α is roughly equal to 0.5, the expected returns are therefore halfway between the two theoretical lines. The upper and lower limits in Figure 4.5 can also be interpreted as the expected returns if $\alpha_i = 0 \; \forall \; i$ (the quoted odds are equal to the expected odds) or $\alpha_i = 1 \; \forall \; i$ (the fair odds are equal to the odds level for the quote above) respectively.

Firstly and most importantly, an upward sloping line in Figure 4.5 would be consistent with returns exhibiting a FL-Bias. A strictly upward sloping line would signify that (4.7) or (4.23) would be satisfied for all odds levels and there would be a strict FL-Bias. This is not the case; the nature of the grid only causes a loose FL-Bias. There is clearly a FL-Bias present due to the nature of the price grid, but its extent is certainly not as severe as the FL-Bias from the actual data in Figure 4.1. For example, for the actual data, expected losses are between 20-40% for prices less than £0.10, whereas in the environment considered, the expected returns (with a = 0.5) in the corresponding range is between 5-15%, see Figure 4.7 for a comparison.

Secondly, the curve in Figure 4.5 is not smooth, it has ‘saw-tooth’ characteristics. It is downward sloping (returns exhibit a reverse FL-Bias) in some portions. These are the sections of the price-grid where (4.7) is not satisfied. For example, the tick-size in odds between prices of 0.1 (odds of 9/1) to $0.\dot{2}$ (odds of 7/2) is constant, (the tick-size is ½ for odds between 7/2 and 9/1; see Table 4.2). As explained earlier, a constant tick-size causes a reverse FL-Bias, this explains why this part of the curve is downward sloping.

The condition for returns to decline at an increasing rate as price falls, (4.24), is satisfied on the whole, but the rate of the decline clearly does not fall at a strictly increasing rate. Having the condition satisfied for the upward sloping parts of the saw-tooth profile of the curve help ensure that returns decline at an increasing rate for the overall picture. However, Figure 4.7 shows that the rate of decline consistent with the environment considered in this chapter is not as dramatic as that for the actual data.

For longshots with price less than 10 pence (*odds > 9/1*), the nature of the price-grid can explain around 7 percentage points (or 23%) of the 30% loss. In the *0.1 < Price < 0.2* range (*4/1 < odds < 9/1*), where there is a constant tick-size, the price-grid can explain around 4 percentage points (or 30%) of the 13% loss. For *0.2 < Price < 0.3,* (*9/4 < odds < 4/1*) the price-grid can explain 2 percentage points (or 33%) of the 6% loss. At price levels greater than this, it is not clear what the actual returns are and the 95% confidence intervals contain the theoretical predictions based on the environment considered in Section 4.4.

The corresponding figure for the pari-mutuel grid (gross quotable odds of 1.01, 1.02, 1.05, 1.10 and then a constant tick-size of 10 pence) is presented in Figure 4.6. As predicted, with the price-grid due to breakage, the result is the opposite to that of the bookmaker price-grid. There is a small strict reverse FL-Bias for prices below 0.8. Returns improve at a constant rate when price declines. The reason why the pari-mutuel curve becomes upward sloping after a price of 0.8 is because the tick-size falls for the remaining three levels of returnable dividends; condition (4.7) or (4.23) is satisfied here. Prices above this level are extremely rare, in terms of the prices quoted by bookmakers, only 58 out of a sample of 39,137 horses from the 2003 flat season in the UK had a price equal to or above 0.8.

FIGURE 4.6
THEORETICAL LIMITS OF BETTOR RETURNS BASED ON THE PARI-MUTUEL PRICE-GRID

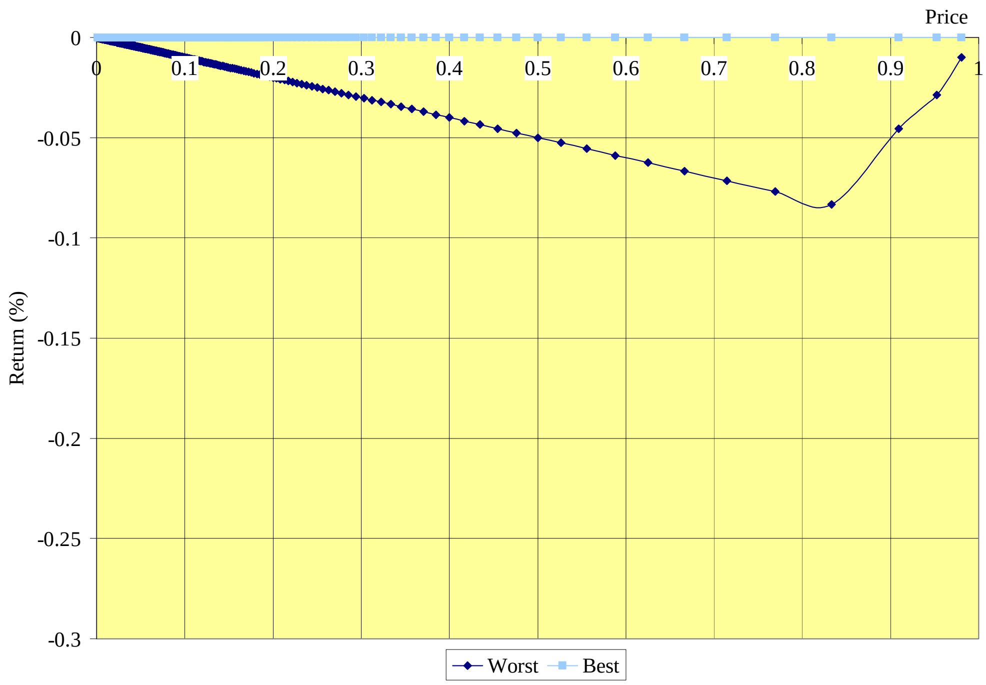
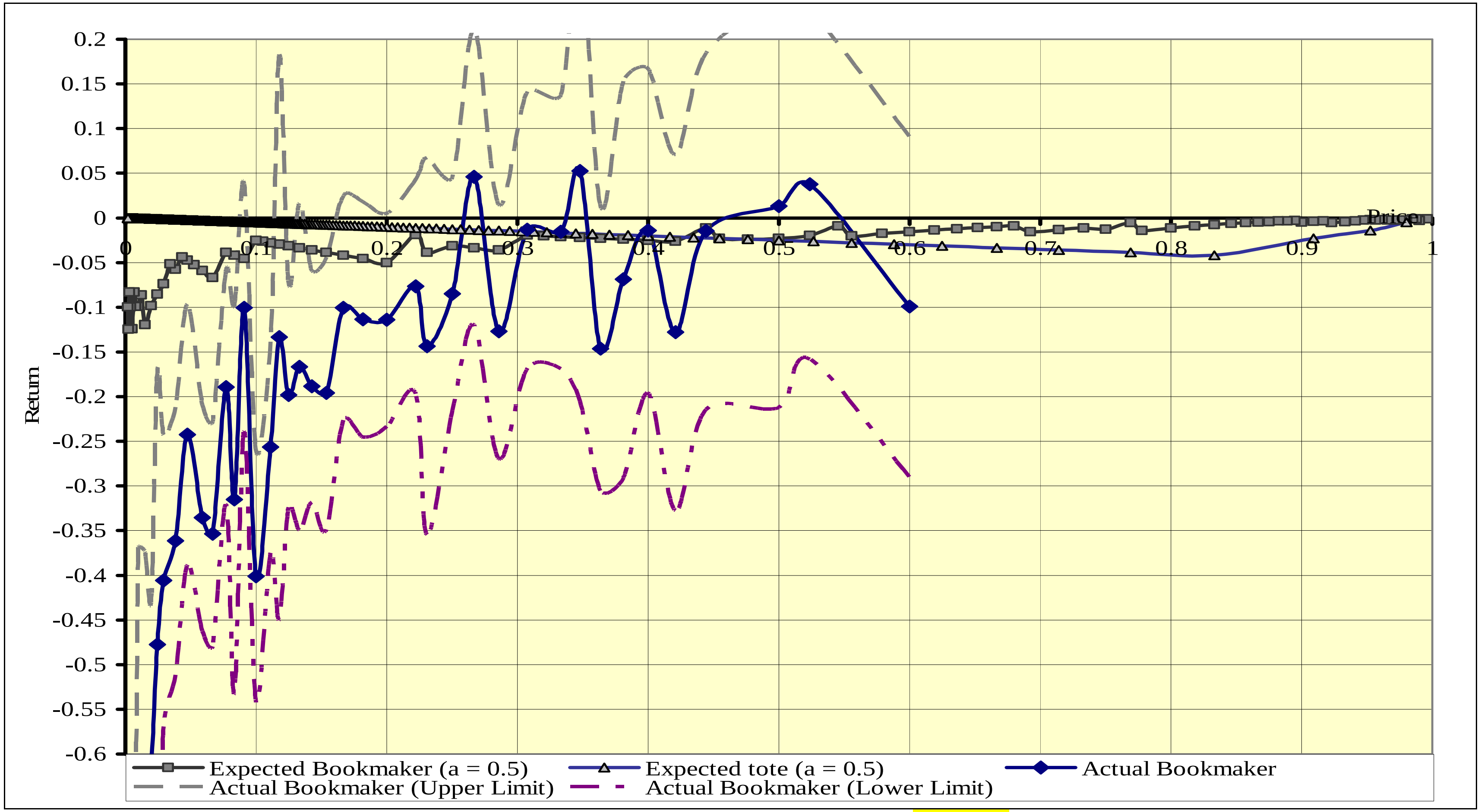


Figure 4.7 provides a comparison of the expected returns from the two grids with *a* = 0.5. It also includes the observed returns from the 2003 data. Unlike Figure 4.1, the points for observed bookmaker returns are not based on classes/ranges of odds, but they are based on the returns of bets placed at every level of quotable odds provided that over 70 observations were recorded at that level for the 2003 flat season. This is accompanied by curves depicting the 95% confidence intervals of the estimate based on the true standard error of returns as in Chapter 2.

FIGURE 4.7
COMPARISON OF THEORETICAL AND ACTUAL BOOKMAKER RETURNS



Note: Actual returns from quoted bookmaker odds displayed (not returns from odds classes as in Figure 4.1). For actual bookmaker returns, upper and lower limits denote 95% confidence intervals (based on the actual standard errors as in Chapter 2) for the observed returns on bets quoted at the respective price.

Expected losses from betting to a pari-mutuel grid cannot be worse than 5%, compared with over 10% when betting on longshots with bookmakers in the environment under consideration. The two curves intersect at a price of approximately £0.50 or odds of evens, this means that given the settings in Section 4.4 and 4.5, bettors should bet with bookmakers when betting on events with odds less than evens, and on the pari-mutuel otherwise. To be more precise, this should only apply to bettors who do not care about knowing exactly what odds they their bets are struck at until after the race has finished. In other words, bettors who do not take the fixed-odds on offer by bookmakers and strike their bets at whatever odds prevail at the start of the race. One could argue that the bookmaker odds quoted during the market phase ought to be inferior because these odds are guaranteed, and hence they should incorporate a premium for the bookmaker. However the premium should disappear at the start of the race when starting odds are recorded.

4.7 – CONCLUSIONS

The effect of the price-grid on the returns of bets at different levels of odds has been investigated. An environment where profit maximising risk neutral bookmakers, who know the true win-probabilities of the runners, vie for bets but must quote odds which are on the price-grid is considered. It is found that for a particular grid to cause returns to yield a favourite-longshot bias, the relative gain in the effective tick-size (of odds), has got to be greater than the relative gain in the gross quotable odds; the tick-size needs to grow as a function of the odds. A price-grid with a constant tick-size, such as that used in pari-mutuel betting, will sway returns towards exhibiting a reverse favourite-longshot bias; this confirms Coleman's (2004) empirical result. This is the first investigation to consider the impact of bookmaker price-grids on the expected returns of bets. The price-grid adopted by British bookmakers, which consists of a narrow tick-size at low odds levels and a wide tick-size at higher levels of odds (though the growth is by no means even), can explain around 10-20% of the losses incurred the bettor for longshots, and causes the bias to be exaggerated.

Given this result, the findings of any investigations of the favourite-longshot bias using odds data should be treated with a little caution since the grid itself is one possible cause of the bias. Also, any comparisons of bettor returns from pari-mutuel markets against bookmaker betting markets using broken odds, such as that of Gabriel and Marsden (1990), are likely to bias the results in favour of bookmakers offering superior returns for favourites and inferior returns for longshots.

4.A.1 – APPENDIX: THE DISTRIBUTION OF ODDS EFFECT

In this section the distribution of odds and its impact is investigated. Figure 4.A.1 shows the distribution of the quoted odds from the 2003 dataset (discussed in Chapter 1) at various scales, and Figure 4.A.2 shows the cumulative distribution. The probability density function of the quoted odds increases until between 5/1 and 10/1 and falls dramatically immediately after.

The distribution of the odds can be approximated as (*f*s are subscripted from hereon in to avoid confusion with exponents):

$$h(\omega_f) = \begin{cases} m\omega_f & for \quad \omega_f < 10 \\ \dfrac{\mathrm{M}}{\omega_f^2} & for \quad \omega_f \geq 10 \end{cases} \tag{4.30}$$

And

$$H(\omega_f) = \begin{cases} \dfrac{m{\omega_f}^2}{2} & for \quad \omega_f < 10 \\ 50m + \dfrac{M}{10} - \dfrac{M}{\omega_f} & for \quad \omega_f \geq 10 \end{cases} \tag{4.31}$$

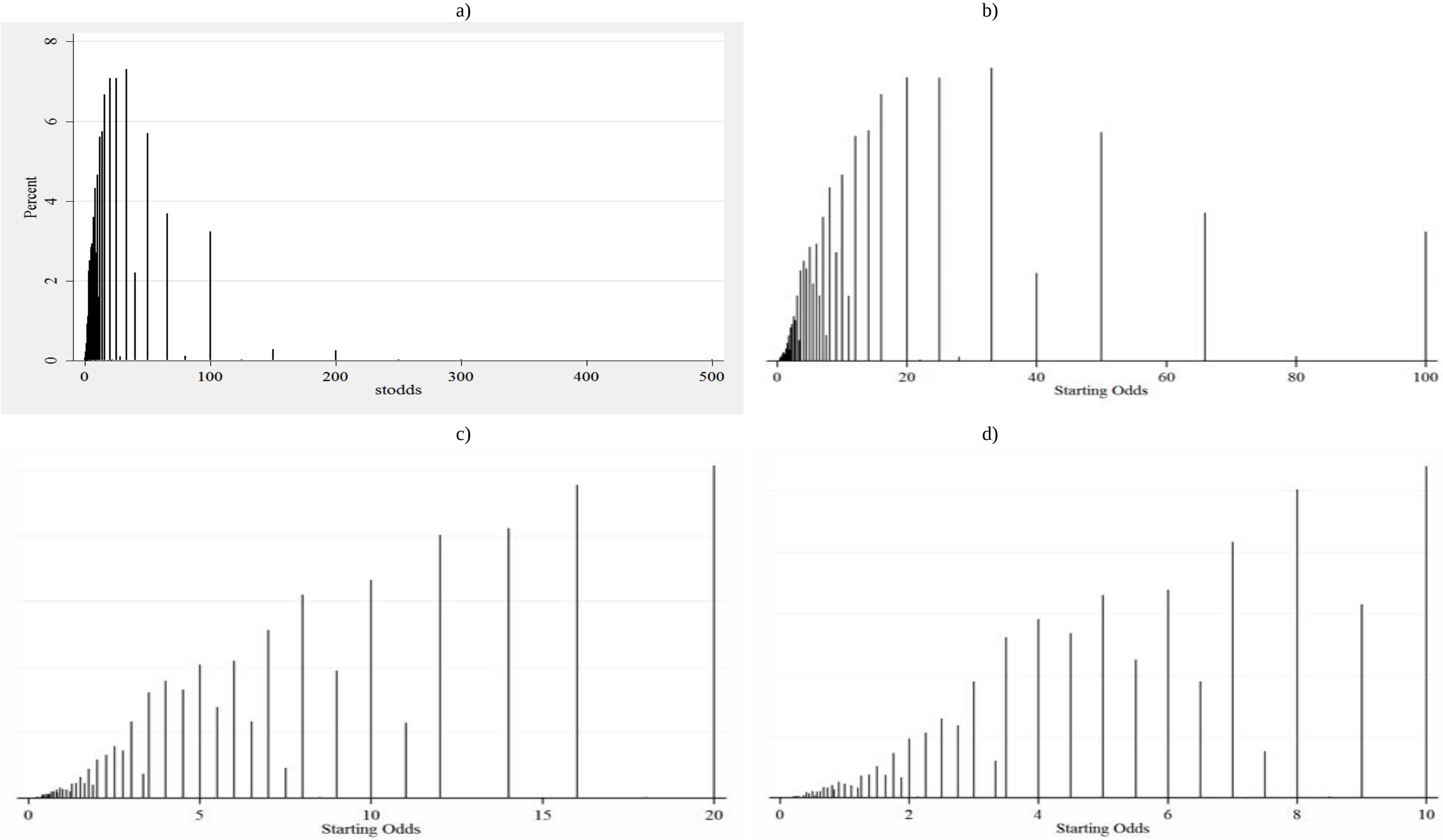
It is found that $M = 5$ and $m = 0.01$ provides a rough fit for the data. Just over 55% of the quoted odds are above 10/1, with $m = 0.01$ the value is 50%. Figure 4.A.3 superimposes h with $M = 5$ and $m = 0.01$ onto panel b of Figure 4.A.1. It is difficult to judge how well the approximation fits the data in this way because of the discrete nature of the quoted odds, especially in the regions where the tick-size is wide.

Using the conditional expectation:

$$h(\omega_f \mid \omega_i \leq \omega_f \leq \omega_{i+1}) = \frac{h(\omega_f)}{H(\omega_{i+1}) - H(\omega_i)} \tag{4.32}$$

FIGURE 4.A.1

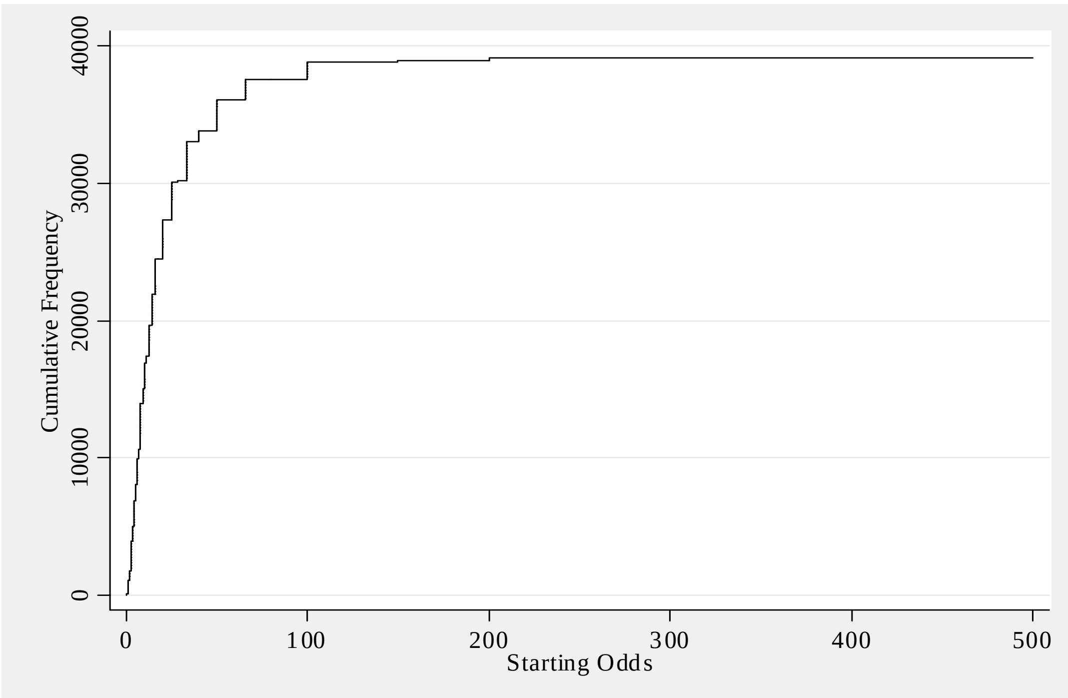
THE DISTRIBUTION OF QUOTED (DISCRETE) STARTING ODDS AT VARIOUS SCALES



Note: The relative frequencies in panels b, c and d are conditional on odds less than the maximum starting odds in the range. Odds are quoted at discrete levels and any unevenness is due to bookmakers not using some levels of these (newer) odds frequently e.g. 80/1.

FIGURE 4.A.2
THE CUMULATIVE DISTRIBUTION OF QUOTED (DISCRETE) STARTING ODDS AT VARIOUS SCALES

a)



b)

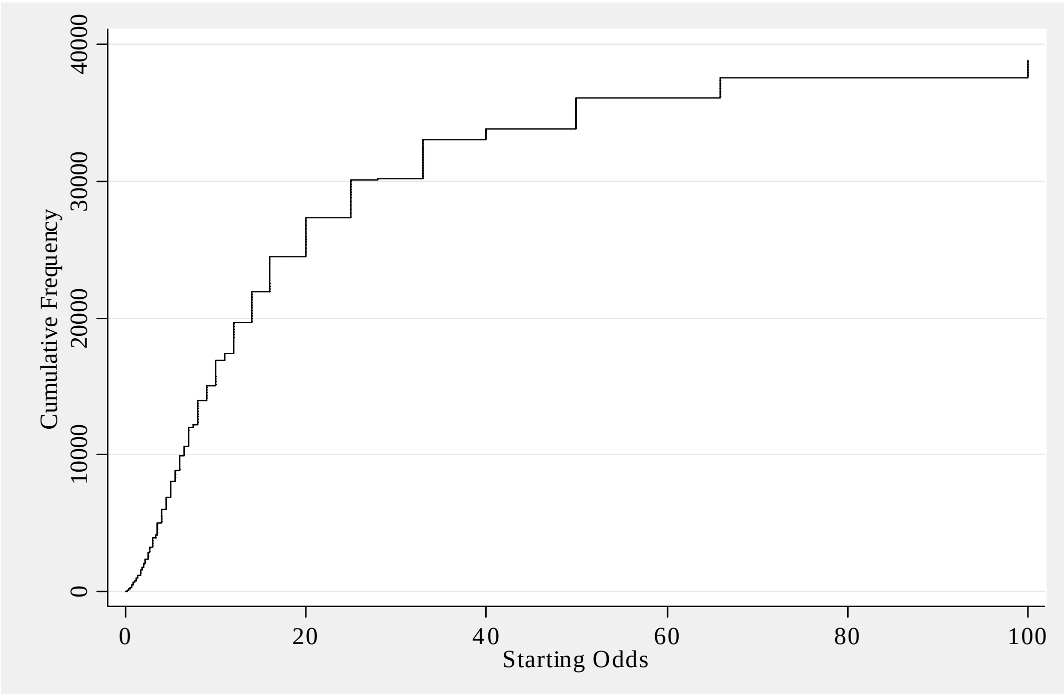
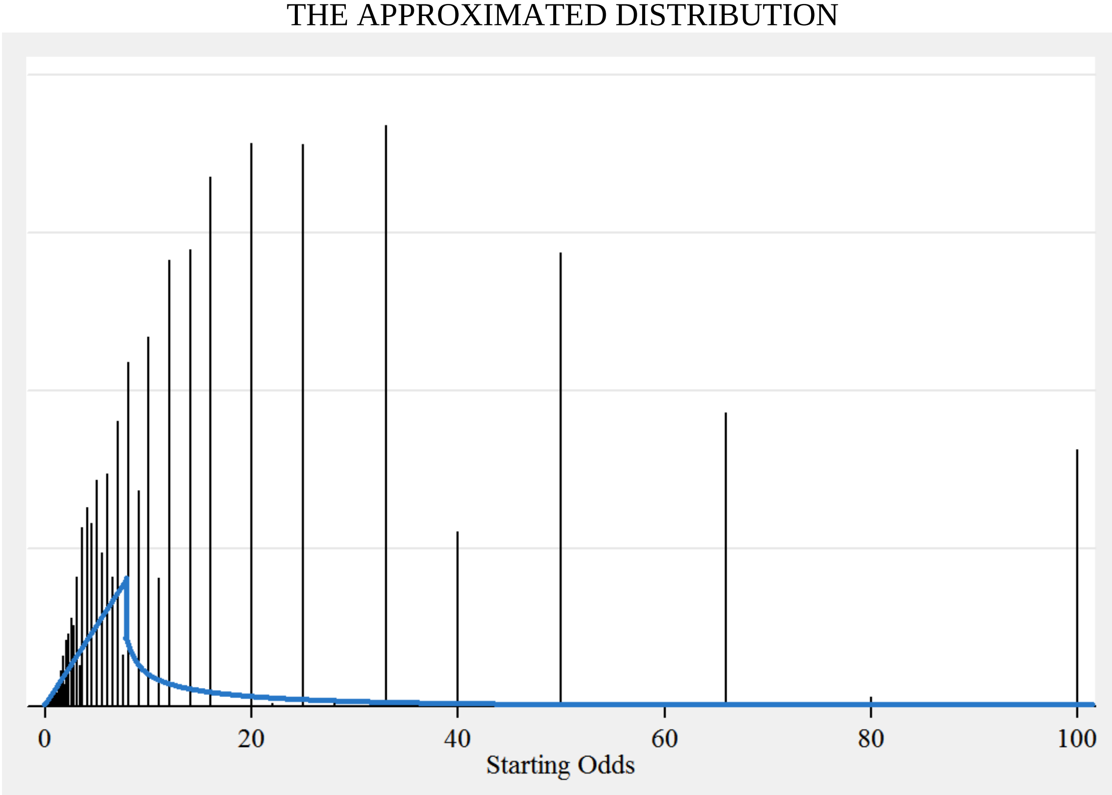


FIGURE 4.A.3
THE DISTRIBUTION OF ACTUAL QUOTED (DISCRETE) STARTING ODDS AND THE APPROXIMATED DISTRIBUTION



If odds of 10/1 are quotable, then for the upward sloping part of the distribution, $\omega^f < 10$, the expected fair odds given the quoted odds are (note, this does not depend on *m*):

$$E(\omega_f \mid \omega_i) = \int_{\omega_i}^{\omega_{i+1}} \omega_f \; h(\omega_f \mid \omega_i \le \omega_f \le \omega_{i+1}) \quad d\omega_f$$

$$E(\omega_f \mid \omega_i) = \int_{\omega_i}^{\omega_{i+1}} \frac{\omega_f (m\omega_f)}{H(\omega_{i+1}) - H(\omega_i)} d\omega_f = \frac{m}{m(\omega_{i+1}^2 - \omega_i^2)\big/2} \int_{\omega_i}^{\omega_{i+1}} \omega_f{}^2 d\omega_f$$

$$E(\omega_f \mid \omega_i) = \frac{2(\omega_{i+1}^3 - \omega_i^3)}{3(\omega_{i+1}^2 - \omega_i^2)} \tag{4.33}$$

For the downward sloping part, $\omega \ge 10$:

$$E(\omega_f \mid \omega_i) = \int_{\omega_i}^{\omega_{i+1}} \frac{M\omega_f}{\omega_f{}^2 [H(\omega_{i+1}) - H(\omega_i)]} d\omega_f = \int_{\omega_i}^{\omega_{i+1}} \frac{M}{\omega_f [H(\omega_{i+1}) - H(\omega_i)]} d\omega_f$$

$$E(\omega_f \mid \omega_i) = \frac{M}{M\left[\frac{1}{\omega_i} - \frac{1}{\omega_{i+1}}\right]} \int_{\omega_i}^{\omega_{i+1}} \frac{1}{\omega_f} d\omega_f$$

$$E(\omega_f \mid \omega_i) = \frac{\omega_i \, \omega_{i+1}}{\omega_{i+1} - \omega_i} \left[\ell n(\omega_{i+1}) - \ell n(\omega_i)\right] = \frac{\omega_i \, \omega_{i+1}}{\omega_{i+1} - \omega_i} \ell n\left(\frac{\omega_{i+1}}{\omega_i}\right), \tag{4.34}$$

which is also independent of M. With the price-grid given in Table 4.2, *a* can be calculated using (4.33) and (4.34). These values are presented in Table 4.A.1.

The values of α given *a* presented in Table 4.A.1 are as governed by expression (4.21). As expected, for the upward sloping parts, *a* > 0.5 and for the downward sloping parts, *a*

< 0.5. The distribution of odds effect caused from the deviation from uniformly distributed odds is small because a is roughly equal to 0.5. It is evident that the values of α are similar to the values of a, and more importantly, α is also roughly constant.

TABLE 4.A.1

CALCULATED VALUES OF *a* AND α GIVEN THE DISTRIBUTION OF QUOTED ODDS AND THE PRICE-GRIDS FOR ODDS AND PRICES

Price	Odds	a	α	Price	Odds	a	α	Price	Odds	a	α
0.9980	1/500	0.5556	0.5560	0.7333	4/11	0.5079	0.5145	0.1667	5/1	0.5079	0.5279
0.9960	1/250	0.5714	0.5729	0.7143	2/5	0.5088	0.5166	0.1538	11/2	0.5072	0.5258
0.9901	1/100	0.5185	0.5191	0.6923	4/9	0.5048	0.5092	0.1429	6/1	0.5067	0.5239
0.9877	1/80	0.5160	0.5166	0.6800	40/85	0.5051	0.5100	0.1333	13/2	0.5062	0.5223
0.9851	1/66	0.5230	0.5242	0.6667	1/2	0.5054	0.5109	0.1250	7/1	0.5057	0.5209
0.9804	1/50	0.5185	0.5197	0.6522	8/15	0.5057	0.5119	0.1176	15/2	0.5054	0.5197
0.9756	1/40	0.5160	0.5173	0.6364	4/7	0.5062	0.5131	0.1111	8/1	0.5051	0.5186
0.9706	1/33	0.5137	0.5150	0.6190	8/13	0.5067	0.5145	0.1053	17/2	0.5048	0.5176
0.9655	1/28	0.5094	0.5105	0.6000	4/6	0.5072	0.5162	0.1000	9/1	0.5088	0.5326
0.9615	1/25	0.5106	0.5119	0.5789	8/11	0.5079	0.5182	0.0909	10/1	0.4841	0.5059
0.9565	1/22	0.5079	0.5090	0.5556	4/5	0.5034	0.5080	0.0833	11/1	0.4855	0.5055
0.9524	1/20	0.5088	0.5101	0.5455	5/6	0.5072	0.5174	0.0769	12/1	0.4867	0.5052
0.9474	1/18	0.5098	0.5114	0.5238	10/11	0.5079	0.5196	0.0667	14/1	0.4885	0.5046
0.9412	1/16	0.5054	0.5064	0.5000	1	0.5079	0.5201	0.0588	16/1	0.4804	0.5082
0.9333	1/14	0.5062	0.5075	0.4762	11/10	0.5072	0.5189	0.0526	18/1	0.4824	0.5075
0.9231	1/12	0.5072	0.5090	0.4545	6/5	0.5034	0.5090	0.0476	20/1	0.4841	0.5069
0.9167	1/11	0.5079	0.5100	0.4444	5/4	0.5079	0.5214	0.0435	22/1	0.4787	0.5093
0.9091	1/10	0.5088	0.5113	0.4211	11/8	0.5072	0.5201	0.0385	25/1	0.4811	0.5084
0.9000	1/9	0.5048	0.5062	0.4000	6/4	0.5067	0.5189	0.0345	28/1	0.4726	0.5124
0.8947	2/17	0.5051	0.5067	0.3810	13/8	0.5062	0.5178	0.0294	33/1	0.4680	0.5147
0.8889	1/8	0.5054	0.5072	0.3636	7/4	0.5057	0.5169	0.0244	40/1	0.4629	0.5174
0.8824	2/15	0.5057	0.5078	0.3478	15/8	0.5054	0.5160	0.0196	50/1	0.4538	0.5219
0.8750	1/7	0.5062	0.5086	0.3333	2/1	0.5051	0.5153	0.0149	66/1	0.4680	0.5154
0.8667	2/13	0.5067	0.5094	0.3200	85/40	0.5048	0.5146	0.0123	80/1	0.4629	0.5180
0.8571	1/6	0.5072	0.5105	0.3077	9/4	0.5088	0.5273	0.0099	100/1	0.4629	0.5181
0.8462	2/11	0.5079	0.5118	0.2857	5/2	0.5079	0.5252	0.0079	125/1	0.4696	0.5149
0.8333	1/5	0.5088	0.5134	0.2667	11/4	0.5072	0.5234	0.0066	150/1	0.4522	0.5235
0.8182	2/9	0.5098	0.5154	0.2500	3/1	0.5088	0.5288	0.0050	200/1	0.4629	0.5183
0.8000	1/4	0.5054	0.5124	0.2308	10/3	0.5041	0.5135	0.0040	250/1	0.4696	0.5150
0.7778	2/7	0.5041	0.5068	0.2222	7/2	0.5057	0.5320	0.0033	300/1	0.4522	0.5237
0.7692	3/10	0.5088	0.5151	0.2000	4/1	0.5098	0.5336	0.0025	400/1	0.4629	0.5185
0.7500	1/3	0.5072	0.5129	0.1818	9/2	0.5088	0.5305				

Note: Values of *a* are calculated using (4.33) and (4.34), α is calculated using the values of *a* and (4.21).