PH4211 STATISTICAL MECHANICS 2002.

Q2 a) Brownian motion is the motion of a <u>macroscopic</u> particle under the influence of microscopic bombardment of many (microscopic) particles. [2]

b) Although this separation was proposed by Langevin is a <u>hypothesis</u>, it may be understood as arising from different centre-of-mass frames of the Brownian particle and the fluid surrounding it. — If the Brownian particle is moving in the fluid then the impacts from the front will be more energetic ~~than~~ those from behind. This will result in a force proportional to (and in opposite direction to) the motion — velocity. [4]

c) From $F = ma$ we get

$$m\frac{dv}{dt} = f(t) - \frac{1}{\mu}v$$

or $$m\frac{dv}{dt} + \frac{1}{\mu}v = f(t)$$

$$\frac{dv}{dt} + \frac{1}{\mu m}v = \frac{f(t)}{m}$$

define $\gamma = \frac{1}{\mu m}$, $A(t) = \frac{f(t)}{m}$ [3]

$$\Rightarrow \frac{dv}{dt} + \underbrace{\gamma\, v(t)}_{\text{friction}} = \underbrace{A(t)}_{\text{random part.}}$$

d) $$v(t) = \underbrace{v(0)\,e^{-\gamma t}}_{\substack{\text{complementary}\\\text{function}\\\text{(CF)}}} + \underbrace{\int_0^t e^{\gamma(u-t)} A(u)\,du}_{\substack{\text{Particular}\\\text{integral}\\\text{(PI)}}}$$

The P.I is the bit that depends on the random force $A(t)$. The CF describes the transient behaviour. [3]

Can get the PI by use of an integrating factor or by use of the system Green's function

e) $\langle A(t)A(t+\tau)\rangle$ describes the average regression of the random fluctuations of $A(t)$.
It may be regarded as a weighted average, where the weight applied to $A(t+\tau)$ is the value at $\tau=0$. [2]

Independent of time is a consequence of the system being in thermal equilibrium. [1]
(stationarity $\equiv$ equilibrium).

f) $$x(t) = \int_0^t v(\tau)\,d\tau$$
Want to look at mean square displacement ~~[illegible]~~

$$\langle x^2(t)\rangle = \int_0^t d\tau_1 \int_0^t d\tau_2 \underbrace{\langle v(\tau_1)\,v(\tau_2)\rangle}_{g_v(\tau_1-\tau_2)}$$

i.e. $\langle x^2(t)\rangle = \underbrace{\int_0^t d\tau_1 \int_0^t d\tau_2 \; g_v(\tau_1 - \tau_2)}_{}$.

change variables $\quad 2\int_0^t (t-\tau)\, g_v(\tau)\, d\tau$ .

At long times put upper limit of integral to infinity. Then.

$$\langle x^2(t)\rangle = 2t \int_0^\infty g_v(\tau)\, d\tau \qquad [3]$$

So $\langle x^2\rangle \propto t$ <u>means diffusion</u>.

In 1d $\quad \langle x^2\rangle = 2Dt$

so $\quad D = \int_0^\infty g_v(\tau)\, d\tau$

— the area under the velocity autocorrelation fn. [2]

20