UNIVERSITY OF LONDON

MSc/MSci EXAMINATION 2012

For Students of the
University of London

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH4211 : STATISTICAL MECHANICS

Time Allowed: **TWO AND A HALF** hours

Answer **THREE** Questions

Approximate part-marks for questions are given in the right-hand margin

The total available marks add up to 120

No credit will be given for attempting any further questions

College Calculators are provided

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	ε_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi\varepsilon_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^{2} kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

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PART MARKS

1. (a) Two isolated systems are brought into contact so that they may exchange energy, but without doing work on each other or exchanging particles between them. Show that the law of entropy increase leads to the concept of *temperature* and that the above systems will reach a state of equilibrium where their temperatures are equal. You must explain what is meant by *equilibrium* in this context. **[10]**

   (b) The derivation in part (a) required that the entropy of the composite isolated system was an *extremum* but there was no need for the extremum to be a *maximum*. Show that the behaviour of the *heat capacity* is influenced by requiring the entropy to be a maximum. **[10]**

   (c) Consider again the two systems in part (a), but now allow them to exchange particles as well as thermal energy. Show that in equilibrium the *chemical potentials* of the systems will become equal. Explain clearly whether particles flow from regions of high chemical potential to regions of low chemical potential or *vice versa*. **[10]**

   (d) A system can exchange thermal energy and particles with a *reservoir* characterized by a temperature T and a chemical potential μ. Show that the probability of funding the system in a microstate with energy E and number of particles N is proportional to $e^{-(E-\mu N)/kT}$. **[10]**

2. (a) Explain what is meant by *phase space* in the context of classical Statistical Mechanics and contrast the Gibbs and Boltzmann conception of phase space. **[8]**

   (b) The density of representative points in phase space is denoted by ρ. Give an expression for the entropy of a classical system in terms of ρ. **[8]**

   (c) Liouville's theorem states that as a system evolves in time the density of points in phase space remains constant. Explain clearly why this theorem is incompatible with the Second Law of thermodynamics. **[8]**

   (d) Give an outline of the resolution of this paradox by the use of *coarse graining* and show how quantum mechanics may be invoked to justify the procedure. **[8]**

   (e) Quantum mechanics is also important in understanding the Third Law of thermodynamics. Explain why this is, and discuss how the Third Law would be stated in a purely classical (non-quantum) world. **[8]**

3. (a) For the case of the ferromagnetic phase transition:

   i) What is the order parameter?

   ii) Is the order parameter conserved or non-conserved?

   iii) What symmetry is broken at the transition?

   iv) Is the broken symmetry continuous or discrete?

   v) What is the order of the transition? **[10]**

   (b) In the Weiss model of this transition it is assumed that the magnetic moments **[18]** are subject to an additional 'mean' magnetic field

$$\mathbf{b} = \lambda\mathbf{M}$$

where **M** is the magnetisation and λ is a constant.

The magnetisation of a *non-interacting* assembly of N spin ½ non-interacting magnetic moments μ is given by

$$\frac{M}{M_0} = \tanh\left(\frac{M_0}{N}\frac{B}{kT}\right)$$

where the saturation magnetisation is $M_0 = N\mu$ and the directions of **M** and the applied magnetic field **B** are parallel.

Show that the Weiss model, in the absence of an external magnetic field, can lead to a spontaneous magnetisation given by

$$\frac{M}{M_0} = \tanh\left(\frac{M_0}{M}\frac{T_\mathrm{c}}{T}\right)$$

where $T_\mathrm{c} = \lambda M_0^2 / Nk$. What is the interpretation of T_c?

(c) Sketch the behaviour of the spontaneous magnetisation as a function of temperature and relate this to the order of the transition. **[4]**

(d) In iron the transition to the ferromagnetic phase occurs at a temperature of 1043 K. Estimate the value of the Weiss field that would be responsible for this. Could the electron dipole magnetic field be responsible? Discuss other possible mechanisms for this field. **[8]**

(*You should take μ for the electron to be* 9.27×10^{-24} A m^2.)

4. Write an essay on 'Brownian Motion'. You should include a clear discussion of how Einstein's interpretation of this phenomenon leads to the conclusion that atoms really exist, and how frictional effects arise from fluctuations. **[40]**

5. (a) In the Landau theory of phase transitions the free energy is expanded in terms of the order parameter. What does the term *order parameter* mean in this context? **[6]**

(b) It is an essential feature of the Landau theory that the free energy expansion is *terminated*. The argument that the order parameter is small may be used to justify why the expansion *may* be terminated. However Landau theory requires that the expansion *must* be terminated. Explain this. **[7]**

(c) What determines the order at which the expansion is terminated? **[5]**

(d) Show that when the Landau theory is applied to the conventional ferromagnetic transition it predicts a *second order* transition and that the order parameter critical exponent β will be ½. **[8]**

(e) It is frequently observed that the β critical exponent takes the value 1/3. Discuss the disagreement between the theory and experiment. **[6]**

(f) When the Landau theory is applied to the *ferroelectric* transition it can lead **[8]** to different behaviour: the transition *may* be *first order*. Give an outline of how this comes about.

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