

Week	Book section	Topic
1	<b>1</b>	<b>The Methodology of Statistical Mechanics</b>
1	<b>1.1</b>	<b>Terminology and Methodology</b>
1	1.1.1	Approaches to the subject
1	1.1.2	Description of states
1	1.1.3	Extensivity and the Thermodynamic Limit
1	<b>1.2</b>	<b>The Fundamental Principles</b>
1	1.2.1	The Laws of Thermodynamics
1	1.2.2	Probabilistic interpretation of the First Law
1	1.2.3	Microscopic Basis for Entropy
1	<b>1.3</b>	<b>Interactions – The Conditions for Equilibrium</b>
1	1.3.1	Thermal Interaction – Temperature
1	1.3.2	Volume change – Pressure
1	1.3.3	Particle interchange – chemical potential
1	1.3.4	Thermal interaction with the rest of the world – the Boltzmannfactor
1	1.3.5	Particle and energy exchange with the rest of the world – the Gibbsfactor
1	<b>1.4</b>	<b>Thermodynamic Averages</b>
1	1.4.1	The Partition Function
1	1.4.2	Gibbs Expression for Entropy
1	1.4.3	Free Energy
1	1.4.4	Thermodynamic Variables
2	1.4.5	The beta trick
8	1.4.6	Fluctuations
	1.4.7	The Grand Partition Function
	1.4.8	The Grand Potential
	1.4.9	Thermodynamic variables
	<b>1.5</b>	<b>Quantum Distributions</b>
	1.5.1	Bosons and Fermions
	1.5.2	Grand Potential for Identical Particles
	1.5.3	The Fermi-Dirac Distribution
	1.5.4	The Bose-Einstein Distribution
	1.5.5	The Classical Limit – The Maxwell-Boltzmann Distribution
2	<b>1.6</b>	<b>Classical Statistical Mechanics</b>
2	1.6.1	Phase Space and Classical States
2	1.6.2	Boltzmann and Gibbs Phase Spaces
2	1.6.3	The Fundamental Postulate in the Classical Case
2	1.6.4	The classical partition function
2	1.6.5	The equipartition theorem
2	1.6.6	Consequences of equipartition
2	1.6.7	Liouville's theorem
2	1.6.8	Boltzmann's H theorem
	<b>1.7</b>	<b>The Third Law of thermodynamics</b>
	1.7.1	History of the Third Law
	1.7.2	Entropy
	1.7.3	Quantum viewpoint
	1.7.4	Unattainability of absolute zero
	1.7.5	Heatcapacity at low temperatures
	1.7.6	Other consequences of the Third Law
	1.7.7	Pessimist's statement of the laws of thermodynamics
2	<b>2</b>	<b>Practical Calculations with Ideal Systems</b>
10	<b>2.1</b>	<b>The Density of States</b>
10	2.1.1	Non-interacting systems
10	2.1.2	Converting sums to integrals
10	2.1.3	Enumeration of states

10	2.1.4	Counting states
10	2.1.5	General expression for the density of states
	2.1.6	Relation between pressure and energy
2	<b>2.2</b>	<b>Identical Particles</b>
2	2.2.1	Indistinguishability
2	2.2.2	Classical approximation
2	<b>2.3</b>	<b>The Ideal Gas</b>
	2.3.1	Quantum approach
2	2.3.2	Classical approach
2	2.3.3	Thermodynamic properties
2	2.3.4	The $1/N!$ term in the partition function
	2.3.5	Entropy of mixing
	<b>2.4</b>	<b>The Quantum Gas</b>
	2.4.1	Methodology for quantum gases
	<b>2.5</b>	<b>Fermi Gas at Low Temperatures</b>
	2.5.1	Ideal Fermi gas at zero temperature
	2.5.2	Fermi gas at low temperatures – simple model
	2.5.3	Fermi gas at low temperatures – series expansion
	2.5.4	More general treatment of low temperature heat capacity
10	<b>2.6</b>	<b>Bose Gas at Low Temperatures</b>
10	2.6.1	General procedure for treating the Bose gas
10	2.6.2	Ground state occupation – chemical potential
10	2.6.3	Number of particles
10	2.6.4	Low temperature behaviour of Bose gas
10	2.6.5	Heat capacity of Bose gas
10	2.6.6	Comparison with Superfluid $^4\text{He}$
	2.6.7	Two-fluid model of superfluid $^4\text{He}$
10	2.6.8	Elementary excitations
	<b>2.7</b>	<b>Quantum Gas at High Temperatures – the Classical Limit</b>
	2.7.1	General treatment for Fermi, Bose and Maxwell Cases
	2.7.2	Quantum energy parameter
	2.7.3	Chemical potential
	2.7.4	Internal energy
	2.7.5	Equation of state
10	<b>2.8</b>	<b>Gas in a harmonic trap</b>
10	2.8.1	Enumeration and counting of states
10	2.8.2	Trapped bosons at low temperatures – Bose-Einstein condensation
	<b>2.9</b>	<b>Black body radiation – the photon gas</b>
	2.9.1	Photons as quantized electromagnetic waves
	2.9.2	Photons in thermal equilibrium – black body radiation
	2.9.3	Planck's formula
	2.9.4	Internal energy and heat capacity
	2.9.5	Black body radiation in one dimension – Johnson noise
	<b>2.10</b>	<b>Ideal Paramagnet</b>
	2.10.1	Partition function and free energy
	2.10.2	Thermodynamic properties
	2.10.3	Negative temperatures
	2.10.4	Thermodynamics of negative temperatures
3	<b>3</b>	<b>Non-ideal Gases</b>
3	<b>3.1</b>	<b>Statistical Mechanics of Interacting Particles</b>
3	3.1.1	The partition function
3	3.1.2	Cluster expansion
3	3.1.3	Low density approximation
3	3.1.4	Equation of state
3	<b>3.2</b>	<b>The Virial Expansion</b>

3	3.2.1	Virial coefficients
3	3.2.2	Hard core potential
3	3.2.3	Square-well potential
3	3.2.4	Lennard-Jones potential
3	3.2.5	The Sutherland potential
3	3.2.6	Comparison of models
3	3.2.7	Universal behaviour
3	3.2.8	Quantum gases – the special case(s) of helium
	<b>3.3</b>	<b>Thermodynamics</b>
	3.3.1	Throttling
	3.3.2	Joule-Kelvin coefficient
	3.3.3	Connection with the second virial coefficient
	3.3.4	Inversion temperature
3	<b>3.4</b>	<b>Van der Waals Equation of State</b>
3	3.4.1	Approximating the Partition Function
3	3.4.2	Van der Waals Equation
3	3.4.3	Estimation of van der Waals Parameters
3	3.4.4	Virial Expansion
	<b>3.5</b>	<b>Other Phenomenological Equations of State</b>
	3.5.1	The Dieterici equation
	3.5.2	The Berthelot equation
	3.5.3	The Redlich-Kwong equation
	<b>3.6</b>	<b>Hard Sphere Gas</b>
	3.6.1	Possible approaches
	3.6.2	Hard Sphere Equation of state
	3.6.3	Virial Expansion
	3.6.4	Virial Coefficients
	3.6.5	Carnahan and Starling procedure
	3.6.6	Padé approximants
4	<b>4</b>	<b>Phase Transitions</b>
4	<b>4.1</b>	<b>Phenomenology</b>
4	4.1.1	Basic ideas
4	4.1.2	Phase diagrams
4	4.1.3	Symmetry
4	4.1.4	Order of phase transitions
4	4.1.5	The order parameter
4	4.1.6	Conserved and non-conserved order parameters
4	4.1.7	Critical exponents
4	4.1.8	The scaling hypothesis
	4.1.9	Scaling of the free energy
4	<b>4.2</b>	<b>First order transition – an example</b>
4	4.2.1	Coexistence
4	4.2.2	van der Waals fluid
4	4.2.3	The Maxwell construction
4	4.2.4	The critical point
4	4.2.5	Corresponding states
	4.2.6	Dieterici's equation
4	4.2.7	Quantum mechanical effects
5	<b>4.3</b>	<b>Second order transition – an example</b>
5	4.3.1	The ferromagnet
5	4.3.2	The Weiss model
5	4.3.3	Spontaneous magnetization
5	4.3.4	Critical behaviour
5	4.3.5	Magnetic susceptibility
5	4.3.6	The ground state and Goldstone modes

5	<b>4.4</b>	<b>The Ising and other models</b>
5	4.4.1	Ubiquity of the Ising model
5	4.4.2	Magnetic case of the Ising model
5	4.4.3	Ising model in one dimension
5	4.4.4	Ising model in two dimensions
5	4.4.5	Mean field critical exponents
5	4.4.6	The XY model
5	4.4.7	The spherical model
6	<b>4.5</b>	<b>Landau theory of phase transitions</b>
6	4.5.1	Landau free energy
6	4.5.2	Landau free energy for the ferromagnet
6	4.5.3	Landau theory – second order transitions
6	4.5.4	Heat capacity in the Landau model
6	4.5.5	Ferromagnet in a magnetic field
6	<b>4.6</b>	<b>Ferroelectricity</b>
6	4.6.1	Description of the phenomenon
6	4.6.2	Landau free energy
	4.6.3	Second order case
6	4.6.4	First order case
6	4.6.5	Entropy and latent heat at the transition
6	4.6.6	Soft modes
7	<b>4.7</b>	<b>Binary mixtures</b>
7	4.7.1	Basic ideas
7	4.7.2	Model calculation
7	4.7.3	System energy
7	4.7.4	Entropy
7	4.7.5	Free energy
7	4.7.6	Phase separation – the lever rule
7	4.7.7	Phase separation curve—the binodal
7	4.7.8	The spinodal curve
7	4.7.9	Entropy in the ordered phase
7	4.7.10	Heat capacity in the ordered phase
7	4.7.11	Order of the transition and the critical point
7	4.7.12	The critical exponent $\beta$
9	<b>4.8</b>	<b>Quantum Phase Transitions</b>
9	4.8.1	Introduction
9	4.8.2	The transverse Ising model
9	4.8.3	Recap of mean field Ising model
9	4.8.4	Application of a transverse field
9	4.8.5	Transition temperature
9	4.8.6	Quantum critical behaviour
9	4.8.7	Dimensionality and critical exponents
7	<b>4.9</b>	<b>Retrospective</b>
7	4.9.1	The existence of order
7	4.9.2	Validity of mean field theory
7	4.9.3	Universality classes
7	4.9.4	Features of Different Phase Transition Models
8	<b>5</b>	<b>Fluctuations and Dynamics</b>
8	<b>5.1</b>	<b>Fluctuations</b>
8	5.1.1	Probability distribution functions
8	5.1.2	Average behaviour of fluctuations
8	5.1.3	The autocorrelation function
8	5.1.4	The correlation time
	5.1.5	Spectral Density – the Wiener-Khinchine Theorem
8	<b>5.2</b>	<b>Brownian Motion</b>

8	5.2.1	Kinematics of a Brownian particle
8	5.2.2	Short time limit
8	5.2.3	Long time limit
8	5.2.4	Equipartition
8	<b>5.3</b>	<b>Langevin's Equation</b>
8	5.3.1	Introduction
8	5.3.2	Separation of forces
8	5.3.3	The Langevin equation
8	5.3.4	Velocity autocorrelation function
8	5.3.5	Mean square velocity and equipartition
8	5.3.6	Diffusion Coefficient
	5.3.7	Harmonically bound particle
	5.3.8	Equipartition and mean-square values
8	5.3.9	Electrical analogue of the Langevin equation
	<b>5.4</b>	<b>Linear Response I – Phenomenology</b>
	5.4.1	Definitions and assumptions
	5.4.2	Response to a harmonic excitation
	5.4.3	Fourier representation
	5.4.4	Response to a step excitation
	5.4.5	Response to a “shock” or delta function excitation
	5.4.6	*Response to a noise excitation
	5.4.7	Consequence of the reality of $X(t)$
	5.4.8	Consequence of causality
	5.4.9	Energy considerations
	5.4.10	Static susceptibility
	5.4.11	Relaxation time approximation
	<b>5.5</b>	<b>Linear Response II – Microscopics</b>
8	5.5.1	Onsager's hypothesis
	5.5.2	Nyquist's Theorem
	5.5.3	Calculation of the step response function
	5.5.4	Calculation of the autocorrelation function
	<b>A</b>	<b>The Gibbs-Duhem Relation</b>
	A.1	Homogeneity of the fundamental relation
	A.2	The Euler relation
	A.3	Two caveats
	A.4	The Gibbs-Duhem relation
	<b>B</b>	<b>Thermodynamic Potentials</b>
	B.1	Equilibrium states
	B.2	Constant temperature and volume: the Helmholtz potential
	B.3	Constant pressure and energy: the Enthalpy function
	B.4	Constant pressure and temperature: the Gibbs free energy
	B.5	Differential expressions for the potentials
	B.6	Natural variables and the Maxwell relations
	<b>C</b>	<b>Mathematica notebooks</b>
	C.1	Chemical potential of a Fermi gas at low temperatures – Sommerfeld expansion
	C.1.1	Setting up the problem
	C.1.2	Mathematica manipulations
	C.2	Internal energy of a Fermi gas at low temperatures – Sommerfeld expansion
	C.2.1	Setting up the problem
	C.2.2	Mathematica manipulations
	C.3	Fugacity and chemical potential of the Fermi, Bose and Maxwell gas at high temperatures
	C.3.1	Setting up the problem
	C.3.2	Mathematica manipulations

- C.4 Internal energy of the Fermi, Bose and Maxwell gas at high temperatures
- C.4.1 Setting up the problem
- C.4.2 Mathematica manipulations
- C.5 Chemical potential of the Bose gas at low temperatures
- C.5.1 Setting up the problem
- C.5.2 Mathematica manipulations
- C.6 Internal energy (and heat capacity) of the Bose gas at low temperatures
- C.6.1 Setting up the problem
- C.6.2 Mathematica manipulations

**D Evaluation of the Correlation Function Integral**

- D.1 Initial domain of integration
- D.2 Transformation of variables
- D.3 Jacobian of the transformation

**E Bose-Einstein and Fermi-Dirac Distribution Functions**

- E.1 Simple derivation
- E.2 Parallel evaluations