PH2130B Mathematical Methods
Course Strategy
Lecturer: Professor B Cowan

Background

Building on the successful introduction of Computer Algebra the first year Skills laboratory, initially using Maple and latterly using Mathematica, it was decided in 1998 to develop this further, with the use of Mathematica in the second year Mathematical Methods course. The intention was to enhance learning and understanding through visualisation and rapid experimentation, while at the same time facilitating problem solving.

The course will have laboratory computer sessions where students will gain hands-on experience, together with more formal lectures where the basic ideas will be described.

Aims and Objectives

The primary aims of PH2130B (as given on the Course-unit Proposal Form) are:

- Ability to solve a range of mathematical problems in physics, particularly those involving differential equations.

- Appreciation of the mathematical representation of physical problems and the physical interpretation of mathematical equations, particularly differential equations.

- Competence in the use of Mathematica as an aid in solving and visualising the above.

After completing the course it is intended that you will feel comfortable and confident when encountering differential equations, you will recognise the essential physics that the equations are "telling you" without necessarily solving them. The traditional methods of solving differential equations will be augmented through the use of Mathematica. You will see that the range of ordinary differential equations you encounter can be classified simply and you should have an appreciation of the importance of symmetry.

Teaching, Learning and Assessment Strategy

1. In the first part of the course in the laboratory sessions students will revise and refresh their familiarity with Mathematica. They will follow programmed scripts taking them through a range of Mathematica's functions and facilities, including the use of *notebooks* and their formatting. This will permit some future exercises to be presented entirely as Mathematica notebooks. During this first part of the course the lecture sessions will introduce students to the range of differential equations encountered in physics, their meaning, and classification.

2. The second part of the course will be devoted to the solution of partial differential equations by the method of separation of variables. Cartesian, cylindrical and spherical polar coordinated will be examined; some of these will be explored

experimentally. Here you will discover the importance of symmetry and its relation to the solution of partial differential equations. The result of separation of variables will be a host of different types of ordinary differential equations. You should discover how the symmetry of the system studied manifests itself in the ODEs you obtain.

3. In the third part of the course you will study some ordinary differential equations in detail. A simple consideration of the physical behaviour of the solutions will lead from simple power series expansions, where you will rediscover sines and cosines (!) to the very powerful method of Frobenius. Here Mathematica will be particularly useful; you will be able to discover for yourselves the properties of some of the "special functions" of mathematical physics. At this stage in the course the lab sessions and the lectures will become more closely linked.

4. The final section of the course puts together what you will have learned in the earlier parts. This will be mainly problem-based. You will be in a position to solve a range of differential equation problems. However Mathematica will give you the power to tackle problems involving the special functions in a remarkably straightforward matter. Here you will solve problems that would have been impracticable without Mathematica's assistance. In this final section of the course the lecture sessions will be more of the nature of problem classes.

Along the way you will learn about orthogonal functions, orthogonal function expansions and the important role that this has in connecting differential equations with matrix equations. This will also provide an entrée to the solution of differential equations by *integral transform* methods; you will thus learn about Fourier and Laplace transforms.

You are expected to make your own notes from the lecture sessions. It will also be helpful to keep records of the laboratory sessions, although there the majority of your work will be in Mathematica notebooks. Nevertheless, it will be beneficial to note down newly learned Mathematica commands, and unusual syntax wherever encountered.

Notebooks of your laboratory sessions will be marked with you in the laboratory so that you will be able to have immediate feedback, know what is expected, and see immediately if/where you are going wrong. This will contribute 30% of the final course mark

Problem sheets are distributed every 2 weeks to (a) reinforce the course material, (b) contribute to assessment and feedback, (c) act as a focus for private study and (d) develop problem solving skills. You should tackle these individually. A coursework mark of 10% is allocated on the basis of the answers submitted. The solutions are discussed in problem classes.

There will be a two hour examination for this course. One compulsory question and another two must be answered from a choice of four. The examination will contribute 60% of the final mark.

The Role of Mathematica

Mathematica has in-built commands to solve differential equations (DSolve[…]). This course is *not* about using such commands for solving differential equations. You will learn about differential equations and *you* will learn how to solve them. Mathematica will be a help along the way in assisting you to do the algebra etc. (for example in eliminating careless mistakes and in doing tedious/repetitive manipulations). Thus Mathematica will serve as an aid to conventional learning.

After achieving this you will learn that Mathematica has "built-in" all the Special Functions of mathematical physics (the solutions to the various differential equations). This will enable you, with Mathematica's help, to work with Bessel functions and Hermite polynomials (for example) just as easily as you now work with sines and cosines.

Finally you can, with all the above behind you, use Mathematica's commands to solve differential equations, but this will now be accompanied by a full understanding of what is going on behind the scenes.