PH2130 Mathematical Methods

Problem Sheet 4

These problems should be done using a combination of paper work and computer work using *Mathematica* as you see fit. Remember presentation is important in your solutions.

1 This question considers two concentric conducting spherical shells. The outer shell is maintained at an electric potential of V_{out} and the inner shell is maintained at a potential V_{in}. In the space between the shells the potential obeys Laplace's equation. Your problem is to find the spatial variation of this potential by solving the appropriate Laplace equation subject to the boundary conditions.

a) Symmetry will dictate which coordinate system you should use. Which one is it?

b) The benefits of exploiting symmetry are great, as some independent variables will vanish from the equations. Which variables vanish? Which variables remain? You should recognise the equation for the remaining variable; what is is?

c) The symmetry of the problem determines the allowed values of the separation variables. What values are allowed?

d) What is the general solution for the potential?

e) Now consider the boundary conditions. Impose these on the general solution to obtain the appropriate solution to this problem.

2 In the lectures (lecture 6) we considered the 2d wave equation and we examined the vibrations of a circular drum. We considered the circularly symmetric solutions and you will recall that these involved the J_0 Bessel function. Your problem is to re-examine this example in the general case where we are not restricted to circularly symmetric solutions.

The *problem* (i.e. the boundary condition) has circular symmetry, but the *solutions* need not exhibit that symmetry away from the boundary.

You should work through the derivation we did in class, but now for the general case. You should find your solutions involve Bessel functions of general integer order J_n, where n arises as the separation constant.

What is the physical meaning of n? You should explain this by sketches. The easiest way to do this is by considering nodes and/or antinodes.

Marks will be given for clarity of presentation.