PH2130 Mathematical Methods

Problem Sheet 3

These problems should be worked out by hand, on paper. You might well use *Mathematica* for doing some of the donkey work. If you do you should explain clearly what you did with *Mathematica*, and why; it is probably unnecessary to submit the entire *Mathematica* notebook.

1 Use the simple power series method to solve the following differential equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 3\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0 .$$

It is probably simplest to derive the recurrence relation for the coefficients by hand. But you can use *Mathematica* to generate the coefficients from the recurrence relation. You can then construct, using *Mathematica*, the power series for the solution up to about the tenth term. You should find the `Collect[ ]` command useful here. Describe clearly all your steps.

You can also solve this differential equation directly as it has constant coefficients. What solutions do you find?

You will find that the two linearly independent solutions found by the two methods are different. This does not matter; you should be able to express one pair as linear combinations of the other. Try this; *Mathematica* can be used for the tedious work.

2 Find a simple series solution for the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{1}{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{4x}y = 0 ,$$

that is, a series of the form

$$y(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

$$\sum_{i=0}^{\infty} a_i x^i .$$

Write out the first few terms of the series expansion explicitly and see if you can interpret it in terms of elementary functions.

You only obtain one solution to the differential equation whereas a second order differential equation must have two solutions. Comment on this.

Marks will be given for clarity of presentation.