PH2130 Mathematical Methods

Problem Sheet 2

1 A metal bar of length L has both ends held at a fixed temperature θ_0. The bar is warmer than this towards the middle; at time $t = 0$ the temperature distribution is

$$\theta(x) = \theta_0 + \theta_1 \sin\frac{\pi x}{L}$$

where x is the distance along the bar.

a) Sketch this initial temperature variation.

The field variable $\Psi(x,t) = \theta(x,t) - \theta_0$ obeys the diffusion equation

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} - \frac{1}{D}\frac{\partial \Psi(x,t)}{\partial t} = 0$$

where D is the thermal diffusion coefficient.

b) Solve the differential equation by the method of separation of variables, to show that the temperature variation follows

$$\theta(x,t) = \theta_0 + \theta_1 \sin\left(\frac{\pi x}{L}\right) e^{-D\pi^2 t/L^2} .$$

c) What is the mathematical form for the temperature variation of the centre point of the rod? Sketch this.

d) Make a plot of $\theta(x,t)$ using *Mathematica* to visualise the evolution of the temperature of the bar from its initial condition. Describe this behaviour in words

You will *not* need to use *Mathematica* in solving this problem, except in the last part. In working through the problem you should lay out your solution as clearly as possible and explain what you are doing at each step. You should follow the procedure of: writing the solution in product form, substituting this into the diffusion equation, identifying the separation constant, writing the resultant ordinary differential equations down, solving them subject to the boundary condition and the initial condition, and finally writing down the full solution.

Marks will be given for clarity of presentation.