PH2130 Mathematical Methods

Problem Sheet 1

The problems on this sheet relate to the introductory handout *First and Second Order Ordinary Differential Equations with Constant Coefficients.*

1 Derive the equation analogous to eq.(1.7) or eq.(1.8), for a *parallel* LCR circuit.

2 By making the substitution $t \to t + \tau$ show that eq.(2.3) does satisfy time translation invariance. (The idea is to show that you end up with a similar equation, but with a different constant *A*.)

3 Does the time evolution $x(t) = Ae^{-bt^2}$ obey time translation invariance? Explain.

4 Show that the particular integral given in eq.(2.5) does indeed satisfy eq.(2.4).

5 Derive the expressions for *C* and φ of eq.(2.10) in terms of the constants *A* and *B* of eq.(2.9). In other words, show that $C = \sqrt{A^2 + B^2}$, $\varphi = -\tan^{-1} B/A$.

6 Prove the first of the two properties of *Q* enunciated in section 2.2.5.

7 Derive the steady state solution given in eq.(2.27).

8 Prove the three properties of *Q* enunciated in section 2.3.4.

9 Derive eq.(2.35). That is, given that $x(t) = f_0 C(\omega)\cos(\omega t + \varphi)$ is the response to $f(t) = f_0 \cos\omega t$, show that $x(t) = f_0 C(\omega)\sin(\omega t + \varphi)$ is the response to $f(t) = f_0 \sin\omega t$. You can do this through a change of time variable such that $\omega t \to \omega t - \pi/2$.

10 By considering a complex voltage $V(t) = V_0 e^{-\mathrm{i}\omega t}$ and the defining properties of the inductor, resistor and capacitor in eqs.(1.6), show that the complex impedance of these components is $\mathrm{i}\omega L$, *R* and $1/\mathrm{i}\omega C$ respectively. It is up to you to adopt a suitable definition of complex impedance.

11 By using *Mathematica*, make plots of the response functions $A(\omega), B(\omega), C(\omega)$ and $\varphi(\omega)$. Investigate graphically and analytically the hypothesis that $B(\omega)$ might be the derivative of $A(\omega)$.