PH2130 Mathematical Methods
Lab 7

This script should keep you busy for the next week and possibly the week after. You should aim to create a tidy and well-structured *Mathematica* Notebook. Do include plentiful annotations to show that you know what you are doing, where you have experimented, and what you have learned. You should then find your notebooks useful to you later on when you are working on other problems.

This exercise is concerned with investigating the mathematics of the quantum and the classical simple harmonic oscillator. You will need to refer to your lecture notes and the web sheets entitled Schrödinger's Equation – 2 The Simple Harmonic Oscillator.

The first part of the exercise is to check some of the material covered in the lectures, using *Mathematica*.

1 Show that the function $e^{-y^2/2}$ satisfies the equation

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}y^2} - y^2\psi = 0$$

in the limit $y \to \infty$. Be sure to check the way the argument was framed in the lectures / notes.

2 Show that the substitution $\psi(y) = H(y)e^{-y^2/2}$ converts the equation

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}y^2} + \left(\varepsilon - y^2\right)\psi = 0$$

into Hermite's equation

$$\frac{\mathrm{d}^2 H}{\mathrm{d}y^2} - 2y\frac{\mathrm{d}H}{\mathrm{d}y} + (\varepsilon - 1)H = 0.$$

3 *Mathematica* knows the Hermite polynomials as `HermiteH[n, x]`. Attempt to demonstrate the orthogonality of the Hermite polynomials using *Mathematica*. You should recall that previously, when considering Legendre polynomials, you constructed a table to do this.

The second part of the exercise is concerned with demonstrating the classical limit of the quantum oscillator. You should work through the description, in the notes, of the classical probability function for the harmonic oscillator. When you understand this you should write a summary in your *Mathematica* notebook. You should then present your own comparison of the quantum and classical cases using plentiful plots.