PH2130 Mathematical Methods
Lab 4

This script should keep you busy for the next two weeks. You should aim to create a tidy and well-structured *Mathematica* Notebook. Do include plentiful annotations to show that you know what you are doing, where you have experimented, and what you have learned. You should then find your notebooks useful to you later on when you are working on other problems.

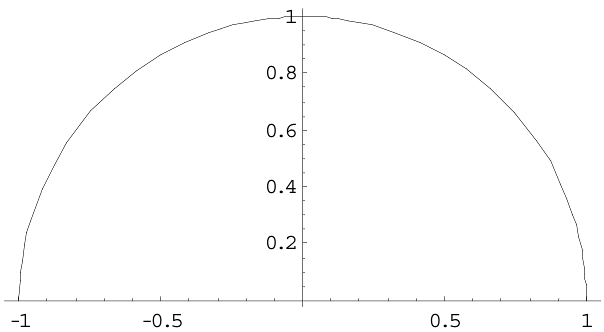
You should have studied the web summary on Orthogonal Functions, and *Mathematica* Notebooks on Fourier Series and on Fourier-Bessel Series. The web pages on Orthogonal functions summarizes what you should have learned in lectures, and it also contains plentiful examples of these series. It includes some diagrams of functions and bar charts of expansion coefficients.

The *Mathematica* Notebooks show how all the calculations were done and how the graphics were produced. You should be familiar with most of these.

Your assignment for the next two weeks is to investigate one Fourier series and one Fourier-Bessel series.

A. Fourier Series

You will study the Fourier series for a semicircle; thus you will see how a semicircle can be constructed from the appropriate superposition of sinusoids.



The semicircle is specified by the function

$$f(x) = \sqrt{1-x^2}\ .$$

1 Plot the function. Since it is supposed to be a semicircle, it will look better if you can get its aspect ratio correct. *Mathematica* has a parameter AspectRatio which can be included in the Plot command. You should investigate using this.

2 You should use the Euler formula to find the Fourier coefficients. Explain why there are only cosine terms.

When you try and evaluate the integrals for the cosine coefficients you might find that *Mathematica* is unhappy about performing the integral over the range $-1 \le x \le 1$.

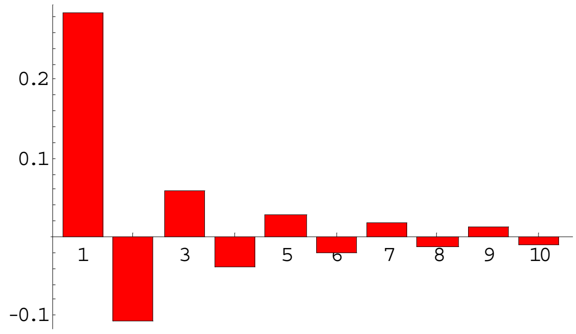
You can get round this by doing the integral in two parts, over $-1 \le x \le 0$ and $0 \le x \le 1$ separately, and then adding the results.

Don't be put off by the Bessel function; *Mathematica* knows all about that even if you don't.

You will need to give the a_0 term special consideration. Just setting $n = 0$ will give an indeterminate answer. Since *Mathematica* is treating n as a variable, you will need to take the limit as $n \to 0$.

You should create a table of the first ten Fourier coefficients (don't put the a_0 term in the table).

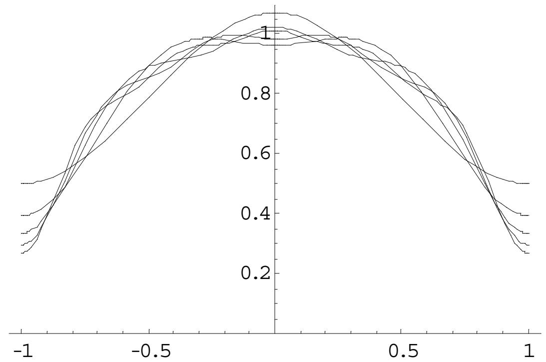
3 Plot a bar chart of the Fourier coefficients. Don't forget to load in the required graphics package.



You should see something like this.

4 Define a function s[m] as the sum of the first m terms of the Fourier cosine series. Don't forget to add the a_0 term here (and remember the extra factor of ½).

5 Plot the Fourier series for various numbers of terms. Comment on how the semicircle is being built up.



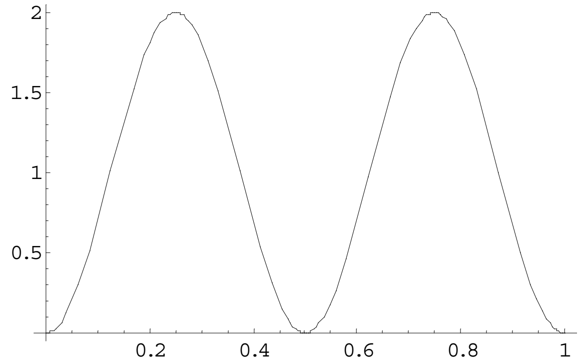
These are some typical examples.

6 Plot the best approximation ($m = 10$) and compare this with the original semicircle.

7 Plot the difference between this approximation and the original function. Comment on the general shape of the error curve.

B. Fourier-Bessel Series

You will study the Fourier-Bessel series for a two sine cycles:



This is specified by the function

$$f(r) = 1 - \cos(4\pi r).$$

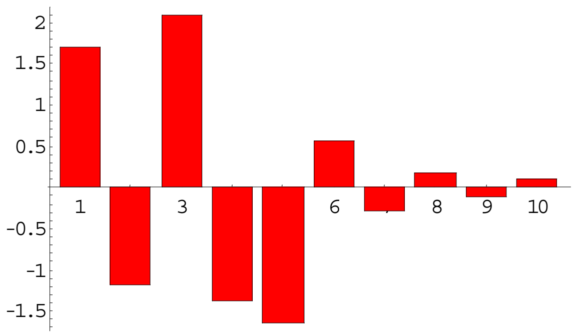
1 Plot the function.

2 In order to evaluate the Fourier-Bessel coefficients you will first need to tabulate the zeros of the J_0 Bessel function. You should follow the procedure adopted in the Fourier-Bessel Series Notebook.

3 You should use the appropriate integral to evaluate the Fourier-Bessel coefficients.

4 Next you should assemble the coefficients into a table. You probably need the first ten coefficients. If you don't find numeric values in the table then you must use the N[ ] command.

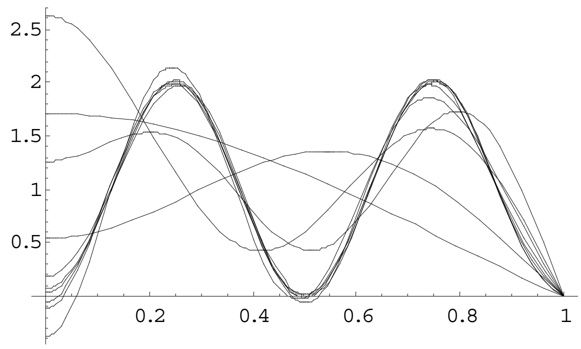
5 Plot a bar chart of the Fourier-Bessel coefficients. If the graphics package has already been loaded in then you don't load it again.



You should see something like this.

6 Define a function s[m] as the sum of the first *m* terms of the Fourier-Bessel series.

7 Plot the Fourier-Bessel series for various numbers of terms. Comment on how the function is being built up.



These are some typical examples.

8 Plot the best approximation ($m = 10$) and compare this with the original function.

9 Plot the difference between this approximation and the original function. Comment on the general shape of the error curve.

When you have completed these two examples you should be familiar with many of the ideas relating to orthogonal functions and the way you can express a given function in terms of them.

You might like to look at the error curves for various approximations. You will see that they are usually wiggly functions and they go through zero quite a lot. Count the number of zeros in these error functions and see how this is related to the number of terms used in the series.